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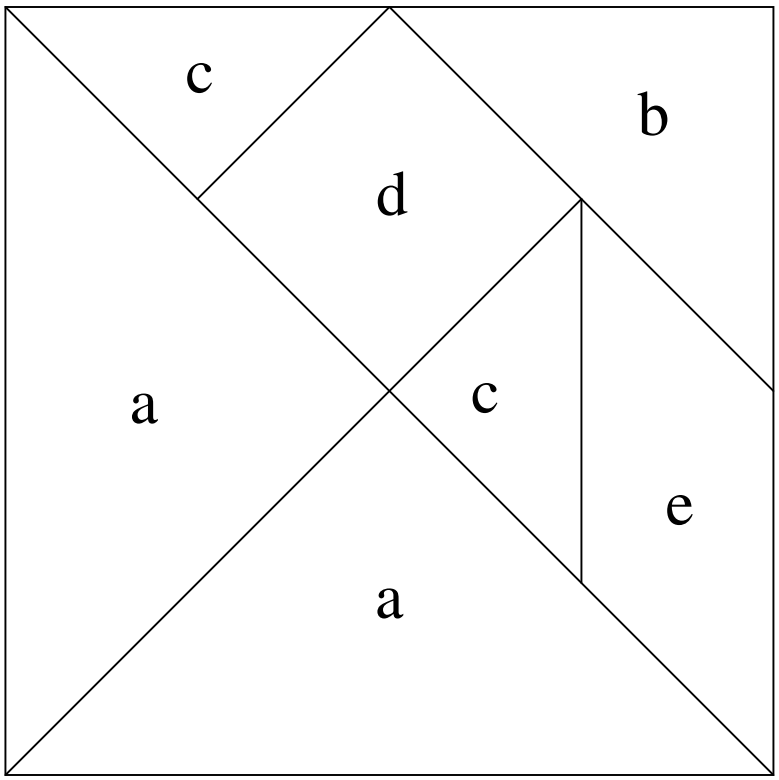
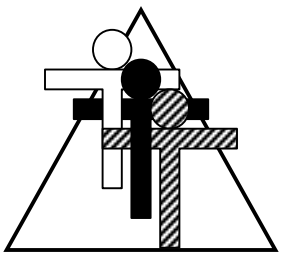


RESOURCE HANDBOOK

By
Brad Fulton and Bill Lombard



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Brad Fulton and Bill Lombard

Teacher to Teacher Press

"*Building Mathematical Skill on a Foundation of Understanding*"

Brad Fulton

- **Consultants**
- **Educators**
- **Authors**
- **Seminar leaders**
- **Teacher trainers**
- **Conference speakers**

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Known throughout the country for motivating and engaging teachers and students, Brad and Bill have authored over ten books that provide easy-to-teach yet mathematically-rich activities for busy teachers. In addition, they have co-authored six teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

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References available upon request.

ACTIVITY 1

Tangram Fractions

Materials:

- ☒ Student copies of tangram masters
- ☒ transparencies of tangram masters
- ☐ scissors

Overview: Students will analyze standard and nonstandard tangrams to determine the fractional values of the pieces. Students will compare the pieces to see how they relate to one another.

Vocabulary: congruent, triangle, rectangle, square, parallelogram, trapezoid, polygon

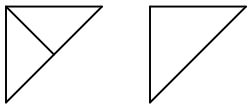
PROCEDURE

Skills:

- Describing and naming polygons
- Studying congruence
- Comparing fractions

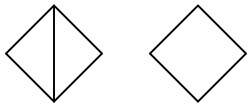
1 This activity works best when students work in groups of two to four. This fosters important dialogue that facilitates understanding. Display a transparency of tangram 1. Students should have individual copies. You may wish to distribute scissors for this activity as some students find it helpful to cut the pieces for comparison.

2 Ask the students, "If the entire square tile has a value of 1, what is the value of the region a?" They can see that it is $^1/_4$ since four of the large triangles can fit in the square.



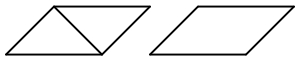
2c = b

3 Next ask them to evaluate the medium-sized triangle, region b. Since two b's will fit into one a, b is $^1/_2$ of a or $^1/_8$ of the whole. Another way to see this is by showing that the tile can be cut into eight b's.

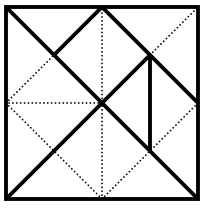


2c = d

4 Ask the students to find the values of the other regions. When they find answers, ask them to justify them. They may do this verbally, by rearranging pieces on their desks or the overhead projector, or by drawing on the image projected onto the board. They will see that the small triangle c is $^1/_{16}$ of the tile since two of them can fit into b. Since two small triangles also fit into the square, d is equal to b. Two c's also fit into the parallelogram e, so b, d, and e are all $^1/_8$. This is shown in the margin.



2c = e



5 Another way to show the relationships among the polygons is by drawing lines to subdivide the tangram into the smallest unit (in this case, c) as shown in the margin. It is then easy to

see that c is $^1/_{16}$ of the tile. Regions b, d, and e are each $^2/_{16}$ or $^1/_8$ of the tile, and region a is $^4/_{16}$ or $^1/_4$ of the tile. This cut-up method will not work on all of the tangram patterns however.

6 Have students explore the other tangram patterns. The first six involve fractions that are based on halves such as $^1/_4$, $^1/_8$, and $^1/_{16}$. Halves are easier to work with conceptually than thirds or fifths. For this reason, only the last two tangrams involve thirds or fifths. Also, in the first three tangrams, all areas use unit fractions; that is, their numerators are one. This changes in tangrams four, five, and six. Again this represents another conceptual step for students.

7 You may wish to have students name each polygon by its characteristics instead of by its letter. For example, in tangram one, region a is the large isosceles right triangle. Regions b and c are also isosceles right triangles. Region d is a square and e is a parallelogram. They can display their answers using these names as shown here.

region	name	fraction
a	right isosceles triangle	¼

Remind them that triangles are always described by their angles and their sides. There are three classifications of each:

angles

acute – all three angles are less than 90º

right – one angle is exactly 90º

obtuse – one angle is more than 90º

sides

equilateral – all three sides are equal

isosceles – two sides are equal

scalene – no sides are equal

Students can check their work by adding all the fractions for the regions of a tangram. These should add up to one (whole tangram).

Journal Prompts:

Explain how polygons b, d, and e are alike and how they are different.

Why does the cut-up method only work on some tiles but not on others?

Homework:

You may wish to assign unfinished tangrams as homework.

Another option is to have students make a tile of their own and write fractions for each region. It should be made of at least seven sections of at least five different shapes. This should be drawn on a 4” or a 6" square. You may wish to pass out grid paper for this task.

Taking a Closer Look:

Rename a region in each tangram equal to one. For example in tangram 1, assume the parallelogram has a value of one. What is the fractional value of each other region?

You may also ask students to find decimal values for each tangram. In addition, you can have students write percent values for each region as shown for tangram 1 below.

Region:	Fraction:	Decimal:	Percent:
a	¼	.25	25%

Assign a cost or value to various regions. If the medium-sized triangle of tangram one sells for $1.37, what is the cost of each region and the whole tile? If tangram two costs $5.64, what would each piece cost?

Assessment:

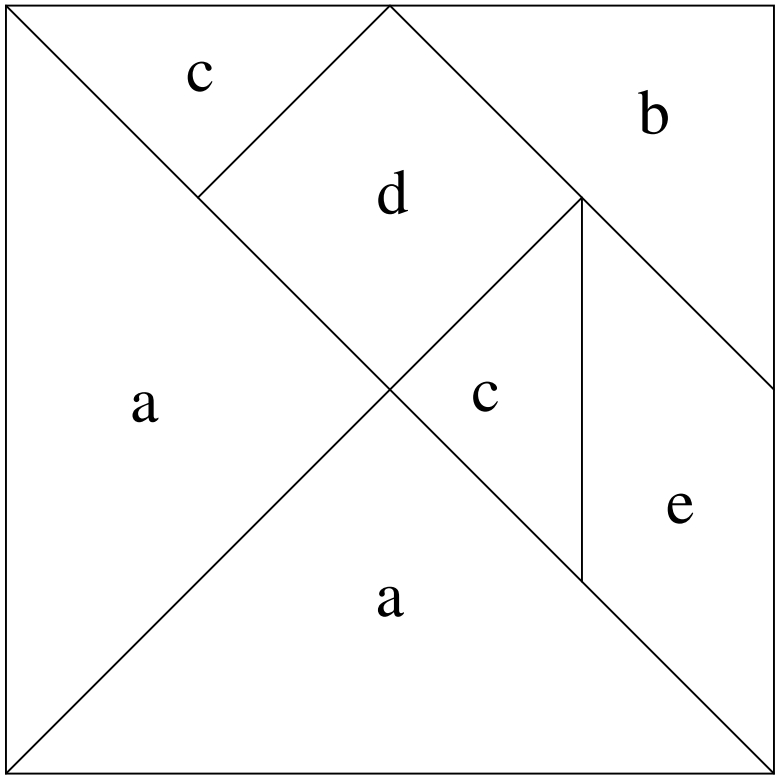
By allowing students to work in groups and by asking them to rationalize their answers, you will be able to assess their levels of understanding.

Homework can be assessed after collecting it, or you may wish to have students trade papers and solve each other’s puzzles.

Answer Key

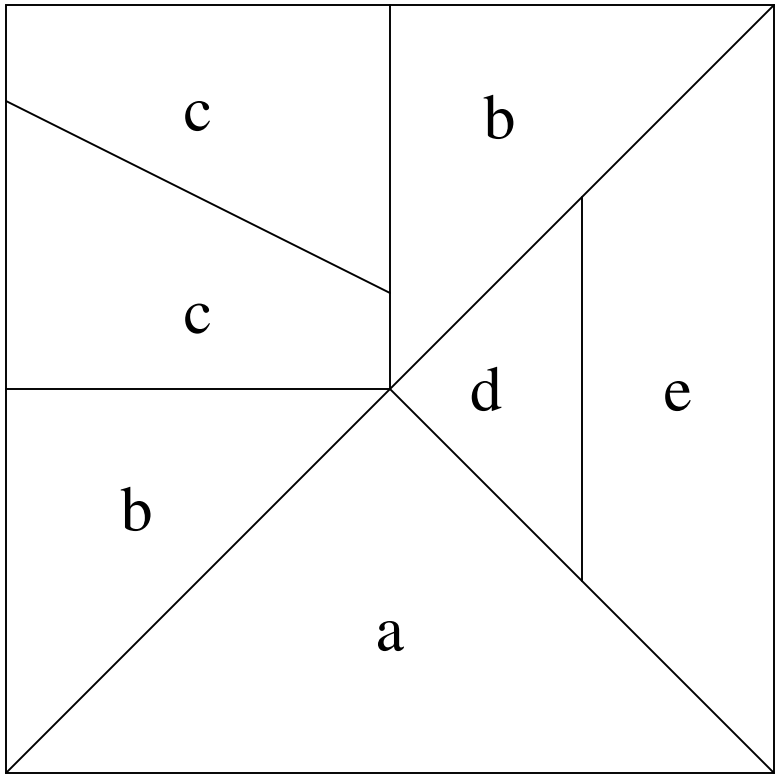
Tangram	Region	Fraction
1	a	$^1/_4$
	b	$^1/_8$
	c	$^1/_{16}$
	d	$^1/_8$
	e	$^1/_8$
2	a	$^1/_4$
	b	$^1/_{16}$
	c	$^1/_8$
	d	$^1/_4$
	e	$^1/_{16}$
	f	$^1/_8$
3	a	$^1/_8$
	b	$^1/_{16}$
	c	$^1/_{16}$
	d	$^1/_{32}$
	e	$^1/_8$
	f	$^1/_4$
	g	$^1/_{32}$
	h	$^1/_{32}$
4	a	$^1/_4$
	b	$^1/_8$
	c	$^1/_8$
	d	$^1/_{16}$
	e	$^3/_{16}$

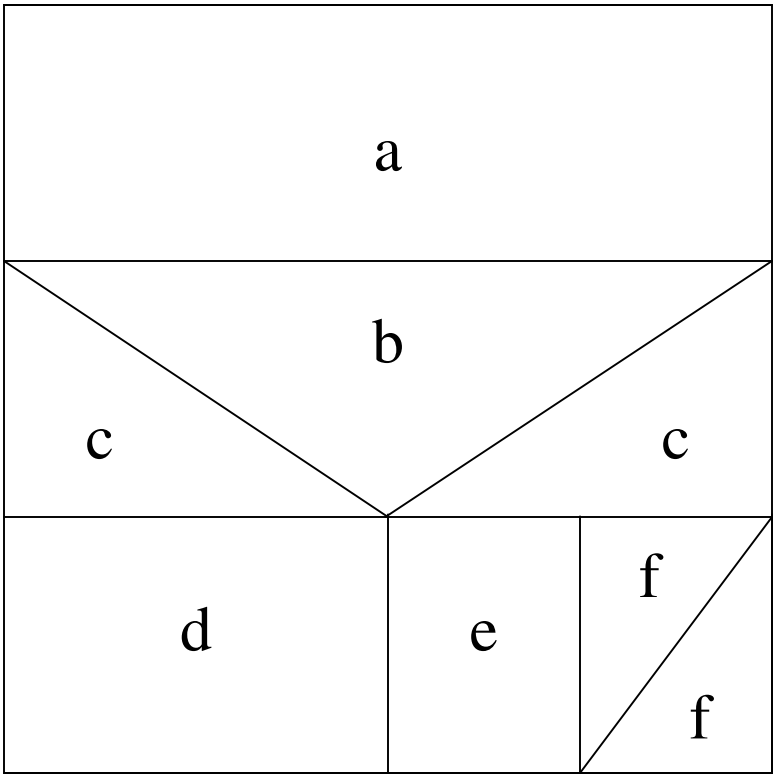
Tangram	Region	Fraction
5	a	$^1/_8$
	b	$^1/_{16}$
	c	$^1/_{16}$
	d	$^1/_{32}$
	e	$^3/_{32}$
	f	$^3/_8$
6	a	$^1/_8$
	b	$^1/_{16}$
	c	$^1/_{32}$
	d	$^1/_{16}$
	e	$^3/_{16}$
	f	$^1/_{16}$
	g	$^1/_8$
	h	$^3/_{16}$
7	a	$^1/_3$
	b	$^1/_6$
	c	$^1/_{12}$
	d	$^1/_6$
	e	$^1/_{12}$
	f	$^1/_{24}$
8	a	$^1/_5$
	b	$^1/_{10}$
	c	$^1/_{20}$
	d	$^1/_{20}$
	e	$^1/_{20}$
	f	$^1/_{10}$
	g	$^1/_{20}$
	h	$^1/_{10}$
	i	$^1/_5$



Tangram 1

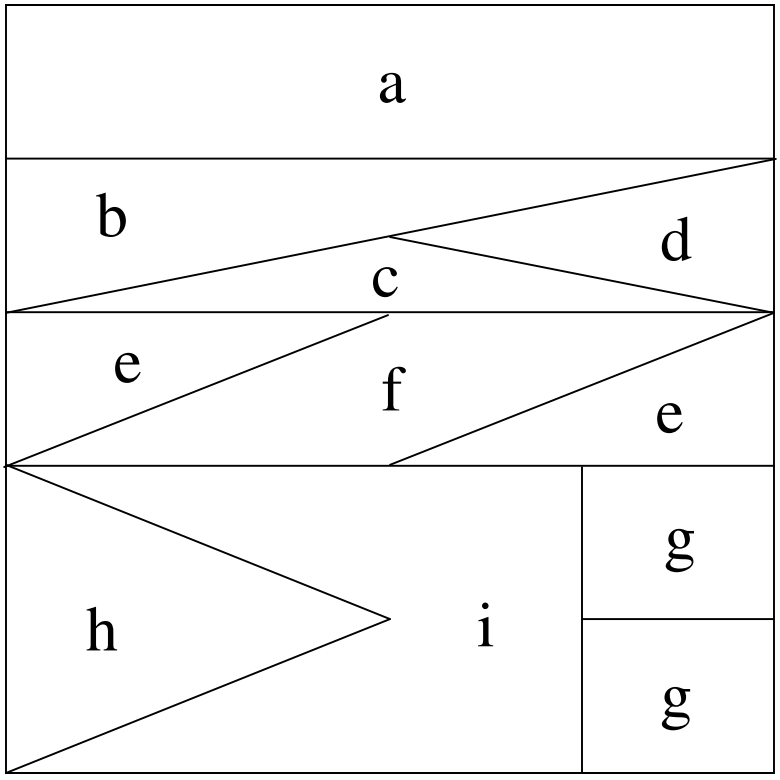
Tangram 4

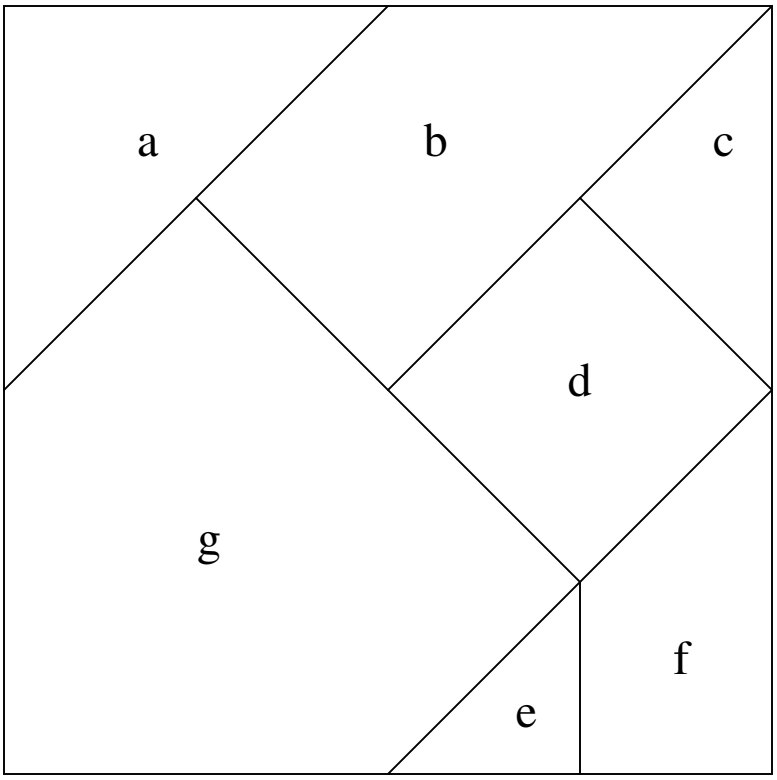




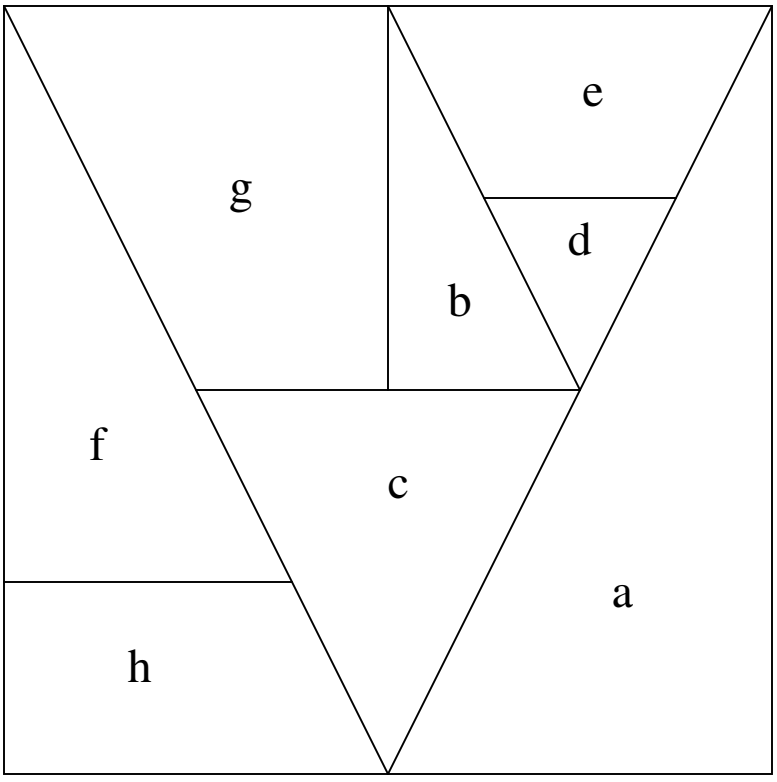
Tangram 7

Tangram 8

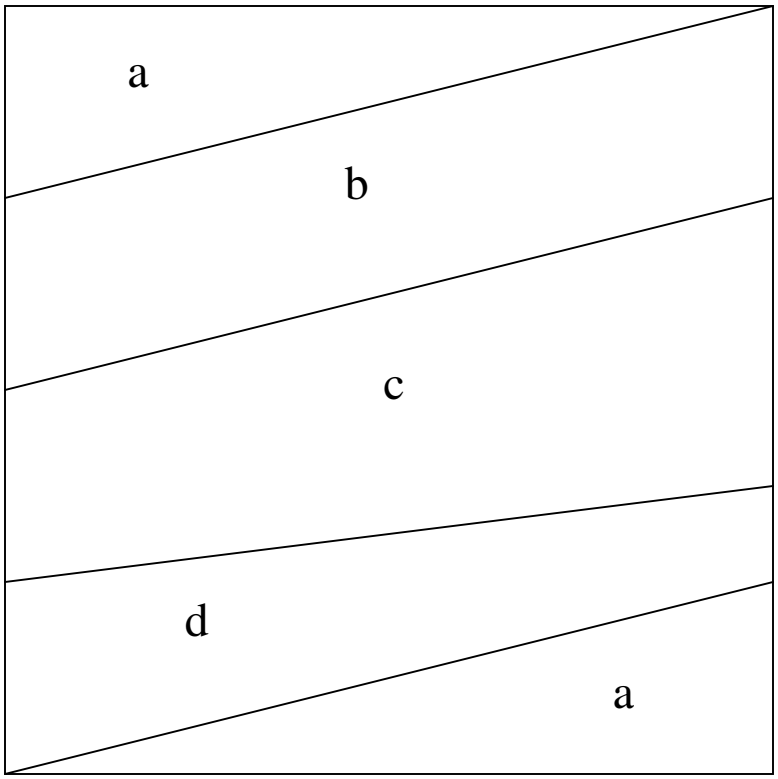




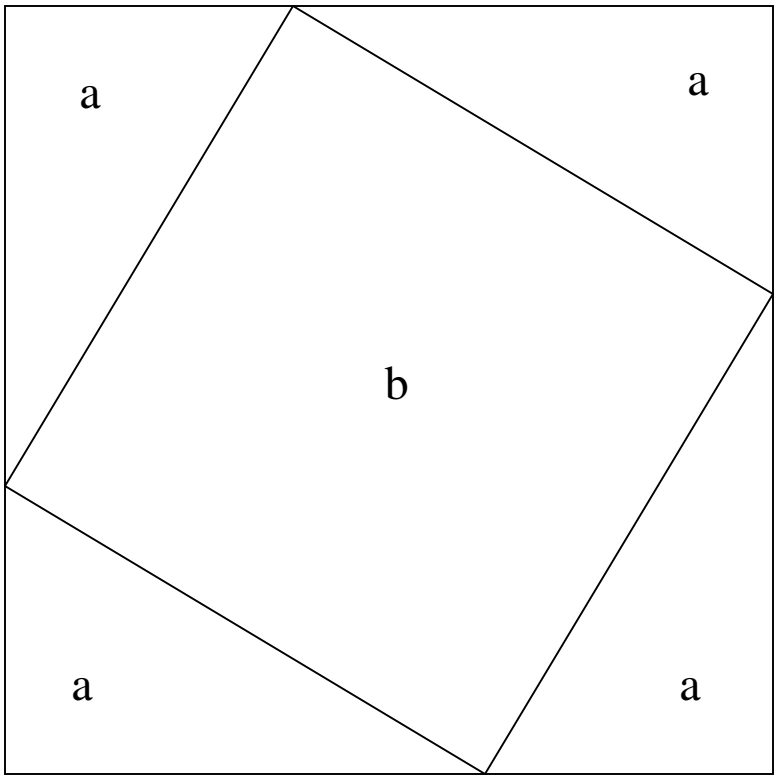
Tangram A



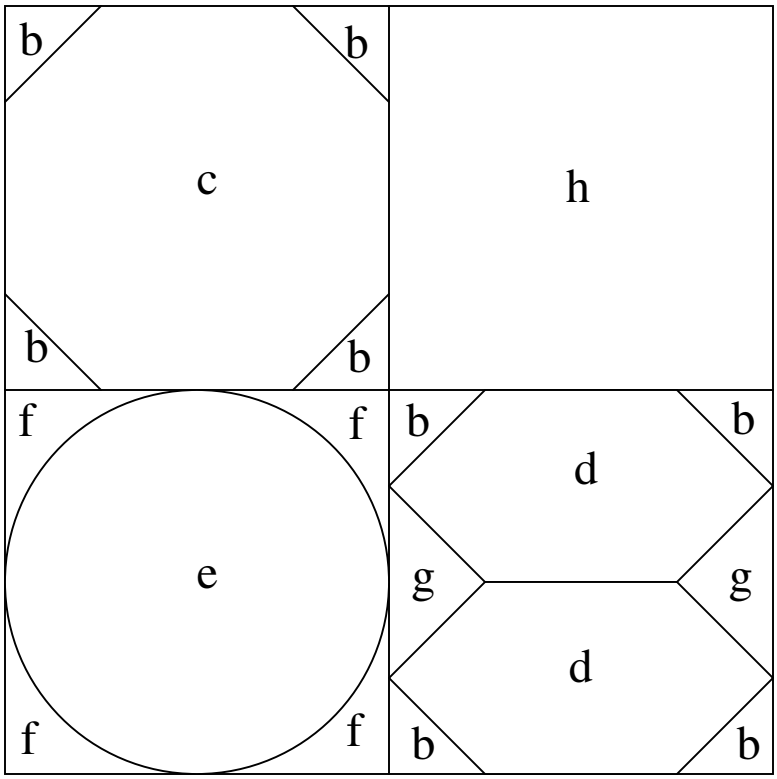
Tangram B



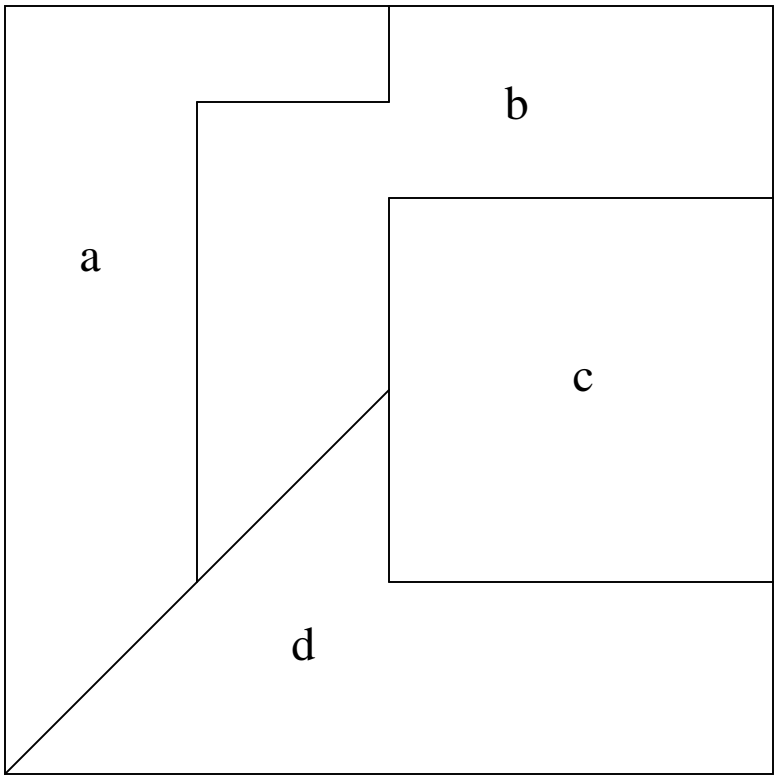
Tangram C



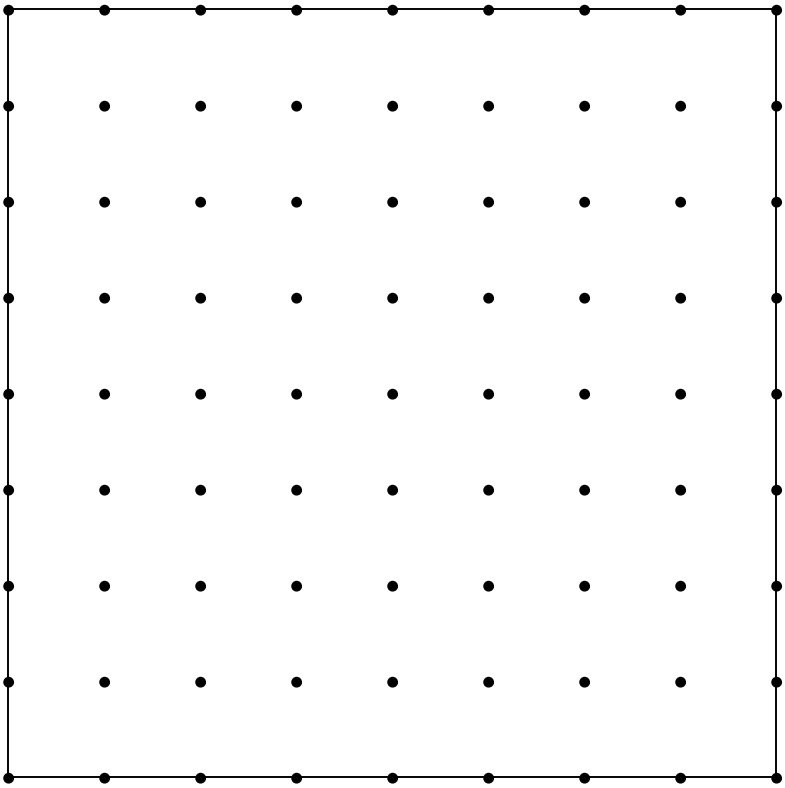
Tangram D



Tangram E



Tangram F



Design your own tangram in the grid provided. Make sure your vertices land on dots.

Name _____

Tangram _____

Region	Fraction	Decimal	Percent	Area	Expression
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____

Tangram _____

Region	Fraction	Decimal	Percent	Area	Expression
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____
_____	_____	_____	_____	_____	_____

ACTIVITY 1

Fair Games 1

Materials:

- ☒ Paper
- ☒ colored chips
- ☐ colored pencils

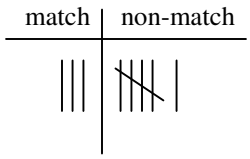
Overview: In this self-motivating investigation students discover the mathematical meaning of fair and unfair. As they work, they will also use reasoning skills, discover patterns, and use Pascal's triangle as a mathematical power tool.

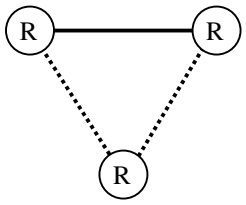
Vocabulary: fair, unfair, theoretical probability, experimental probability

PROCEDURE

Skills:

- Finding probability
- Using probability notation
- Understanding probability
- Finding patterns
- Simplifying fractions



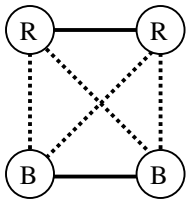


1. Explain to the class that you are going to play a game. In this game two red chips and one blue chip will be placed in a can. Two will be drawn at random. If they match, the students get one point; if they don't match, the teacher gets a point. Ask them who they think will win? Also ask them to explain their reasoning. (Some younger students often reason that since there are two matching red chips but only one blue chip the students will win twice as many games.)
2. Play 20 rounds of the game keeping track of matches and non-matches on a tally chart as shown. The students will be surprised to discover that the non-match occurs about twice as many times as the match. Ask them to try to explain why this occurs. How can the game be made fair? Suggest that they consider putting two red chips and two blue chips in the can.
3. Play 20 rounds of this new game to see if it appears to be fair. Again the teacher will likely win by a considerable margin. Why does this happen?
4. Display transparency master A or sketch it on the board. Have them make a sketch in their notes. Draw connecting lines between chips to record matches and non-matches. It is best visually to use two different colors. Alternatively you can use solid lines for matches and dotted lines for non-matches. The result is shown in the margin. This shows that there are two ways to achieve a non-match and only one way to make a match. It is much easier to see why the teacher will probably win this game. Of the three ways that these chips can be drawn, one of the three results in a match, and two of the three result in a non-match. This means that a

player of this game will *probably* get a match one out of every three games played and a non-match two out of every three games played. Mathematicians have developed a very convenient shorthand way to write this:

$$P(\text{match}) = {}^{1}/_{3} \qquad P(\text{non-match}) = {}^{2}/_{3}$$

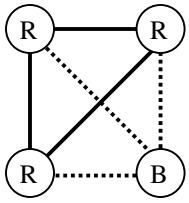
When writing probability the denominator indicates the total possible outcomes. The numerator indicates the total number of outcomes that result in the desired condition (a match or a non-match).



5. Now display transparency master B and have the students sketch this problem in their notes. Have them try to find the probability of a match and non-match. The result is shown in the margin. They will likely be surprised to discover that the outcome is the same as problem A.

$$P(\text{match}) = {}^{2}/_{6} = {}^{1}/_{3} \quad P(\text{non-match}) = {}^{4}/_{6} = {}^{2}/_{3}$$

6. The probability should be written on the transparency master exactly as shown above. It should be noted that probability is written as a common fraction in simplest terms. It is usually not written as a decimal or percent.



7. Ask the students to suggest a game that might be fair. Transparency masters are provided for most situations, but you may have to sketch some that the students suggest. Let's explore problem C. Most students will think the class has an advantage in this game since there are three red chips and only one blue chip. When they play the game, the score will *probably* be much closer than they expect. Once they analyze the game, they will see why that this is a fair game.

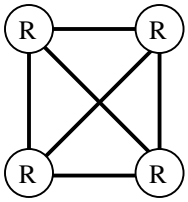
$$P(\text{match}) = {}^{3}/_{6} = {}^{1}/_{2} \quad P(\text{non-match}) = {}^{3}/_{6} = {}^{1}/_{2}$$

8. Some students will think this game is fair *if they win* and unfair if the teacher wins. This is a good time to introduce the mathematical definition of fair. A game is "fair" if each team has an *equal probability of winning*. With two teams that translates into a probability of one half. This does not mean that the game will always result in a tie. It simply means that if enough games are played, there will *probably* be a nearly equal number of both possible outcomes.

9. This leads into the difference between theoretical probability and experimental probability. Let's assume that in game C, there were 12 matches and 8 non-matches in 20 attempts. The *experimental probability* is shown here:

   P(match) = $^{12}/_{20} = {}^{3}/_{5}$ P(non-match) = $^{8}/_{20} = {}^{2}/_{5}$

   However, the *theoretical probability* is still one half for each outcome. Experimental probability is what actually occurs when the experiment in conducted. Theoretical probability is what is expected to occur in the long run given a reasonable amount of experiments.

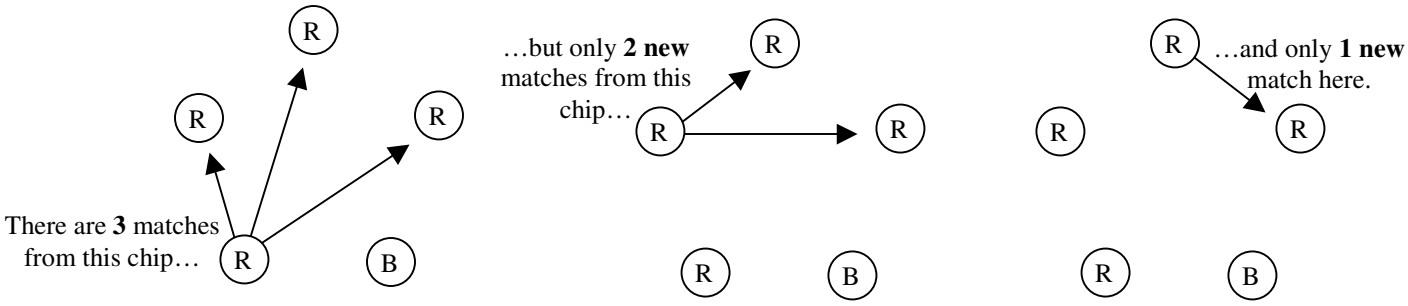


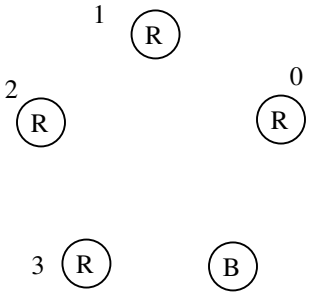
10. To make a point, have the students analyze problem D. Although it is obvious that every game is a match, this situation illustrates two important points.

    P(match) = $^{6}/_{6} = 1$ P(non-match) = $^{0}/_{6} = 0$

    A probability of one is an absolute certainty. A probability of zero is an absolute impossibility. That is why a probability is always written as a fraction greater than or equal to zero and less than or equal to one.

11. Have students explore other games with the goal of finding another fair game. A homework master is provided.





The diagram to the left shows that there are six total matches (3 + 2 + 1 + 0). Since we found out that there are four non-matches (4 red times 1 blue), there must be ten total outcomes. Thus we can write:

P(match) = $^{6}/_{10} = {}^{3}/_{5}$ P(non-match) = $^{4}/_{10} = {}^{2}/_{5}$

This method allows us to analyze larger games without drawing and counting so many lines.

Journal Prompts:

What does fair mean? How has your definition of that term changed during this activity?

How has your definition of the word "fair" changed during this activity? What does "fair" mean to a mathematician?

Homework:

A homework master of sample problems is provided.

Taking a Closer Look:

Many extensions are explored in the next chapter.

Ask students to describe patterns they see. This will help them analyze games of many chips. Some students may notice that the number of possible non-matches is the product of the number of the two colors. For example in game E, there are four red chips and one blue chip. Therefore there will be 4 x 1 possible non-matches. There is also a trick to finding the matches without drawing all the lines. This is shown below.

Assessment:

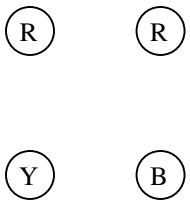
Check to see that students are writing the correct notation for probability. Avoid the temptation to write, "P(m)." This means, "The probability of getting an m." Also see if they are simplifying fractions. A student may have noticed that the probability of the match and the probability of the non-match will always add up to one.

Invite students to the board or overhead projector to solve problems. This allows you to assess their work and allows the class to see any shortcuts they are using.

Homework: Fair Games 1 Name_____

Analyze each game shown to determine the probability of a match and the probability of a non-match. Use different colors to show matches and non-matches. Remember to simplify all fractions.

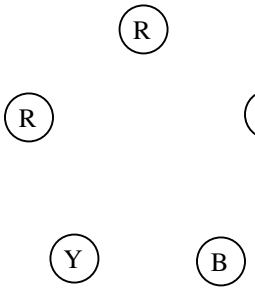
1



P(match) = _____

P(non-match) = _____

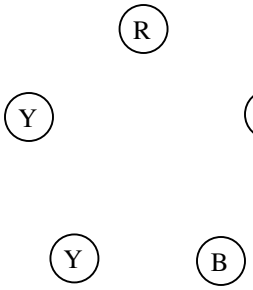
2



P(match) = _____

P(non-match) = _____

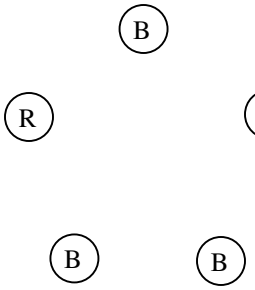
3



P(match) = _____

P(non-match) = _____

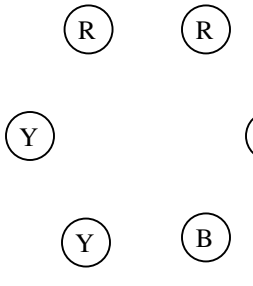
4



P(match) = _____

P(non-match) = _____

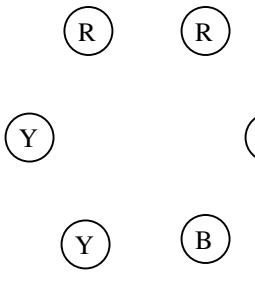
5



P(match) = _____

P(non-match) = _____

6



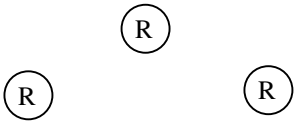
P(match) = _____

P(non-match) = _____

Homework: Fair Games 2 Name_____

Analyze each game shown to determine the probability of a match and the probability of a non-match. Use different colors to show matches and non-matches. Remember to simplify all fractions.

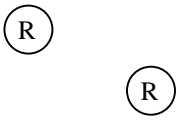
1



P(match) = _____

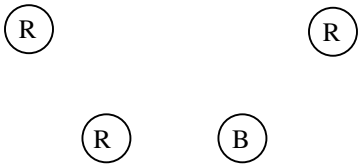
P(non-match) = _____

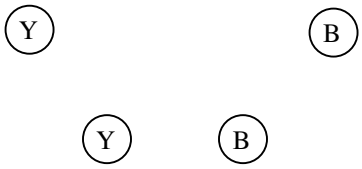
2

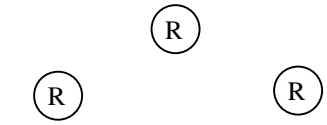


P(match) = _____

P(non-match) = _____



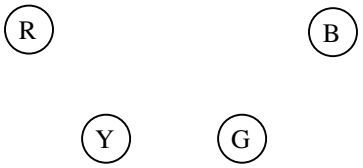


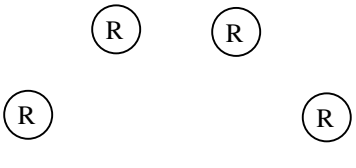


3

P(match) = _____

P(non-match) = _____





4

P(match) = _____

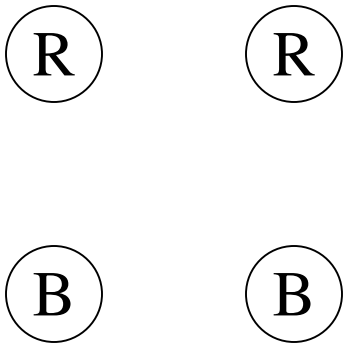
P(non-match) = _____

5

P(match) = _____

P(non-match) = _____

The following chips are to be placed in a can. Two will be drawn at random. What is the probability of a match? What is the probability of a non-match?



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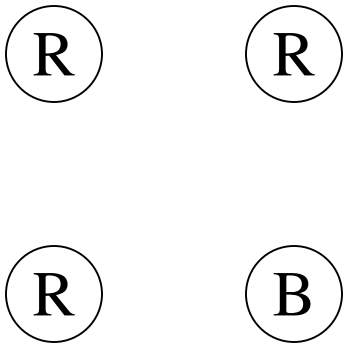

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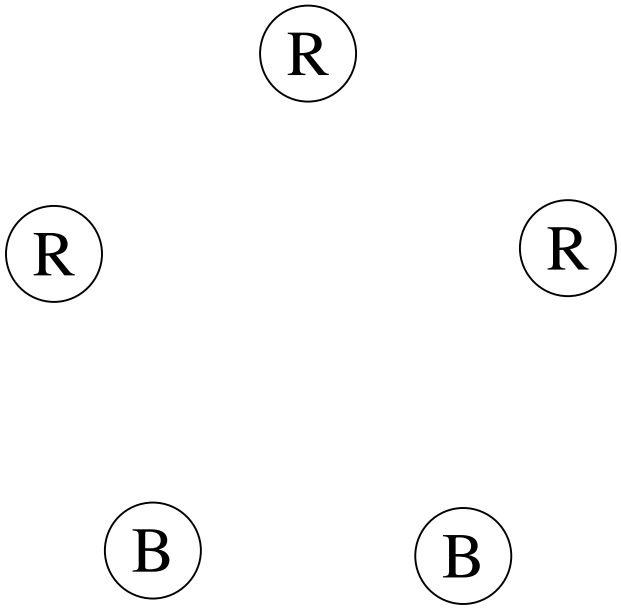

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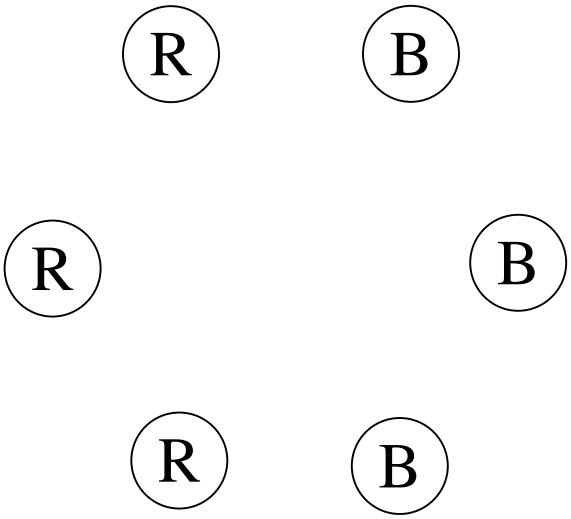

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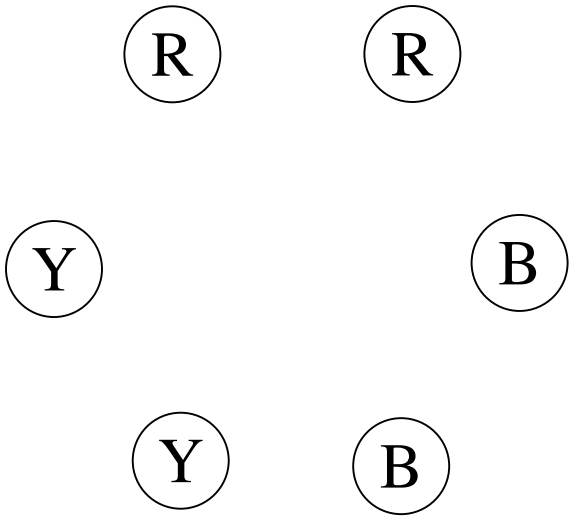

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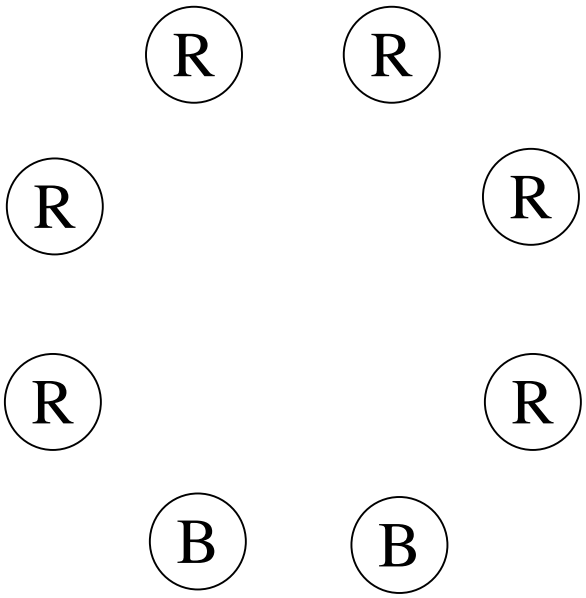
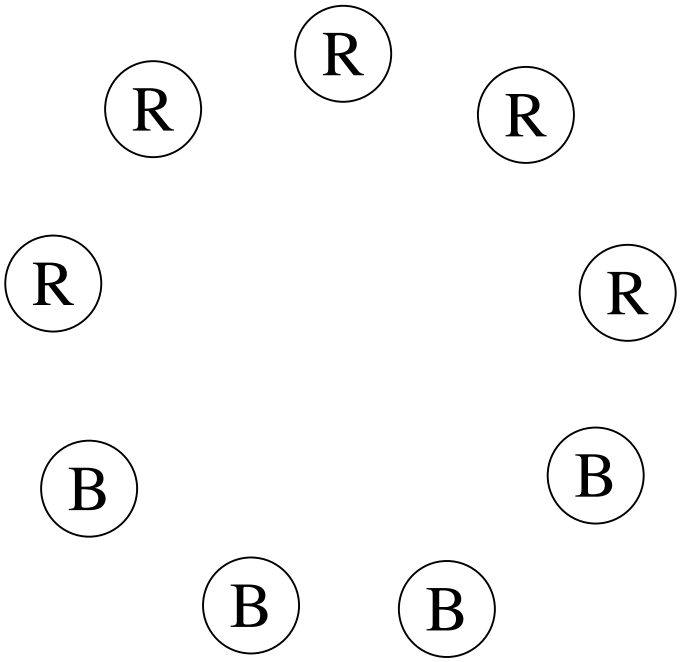


The following chips are to be placed in a can. Two will be drawn at random. What is the probability of a match? What is the probability of a non-match?



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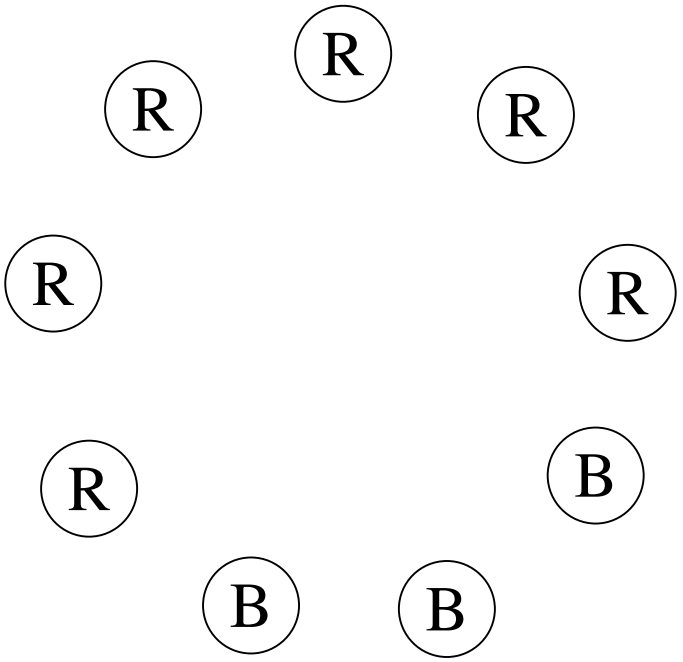

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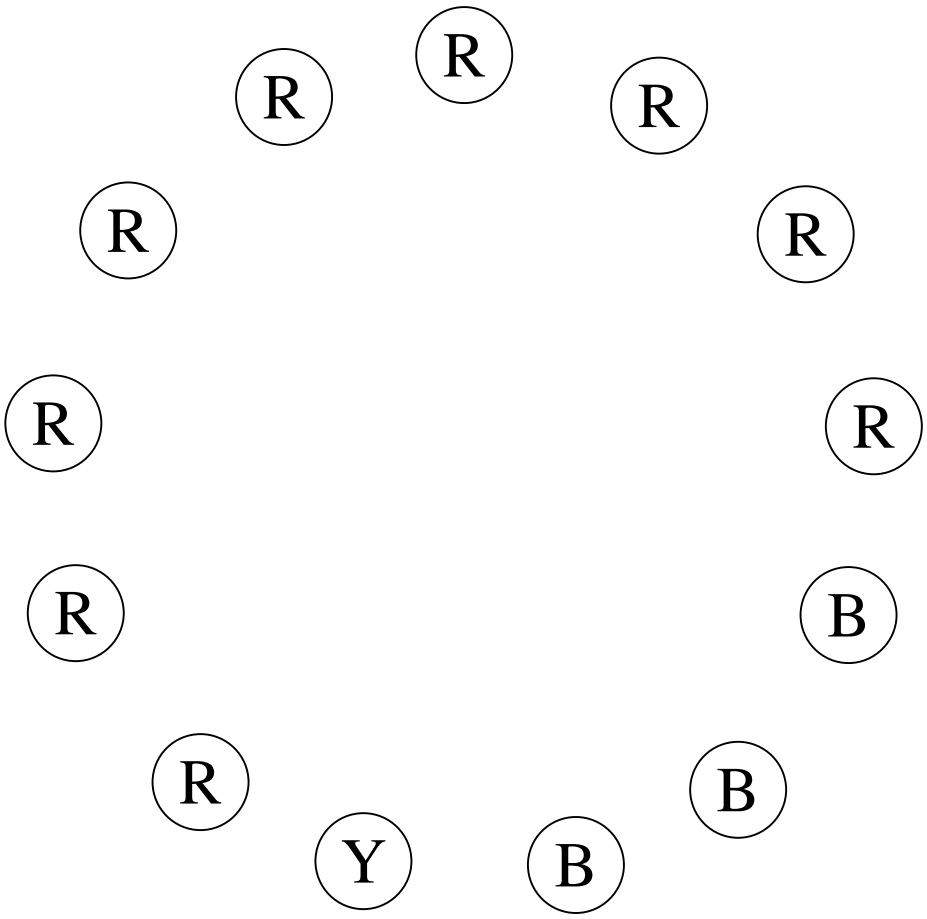

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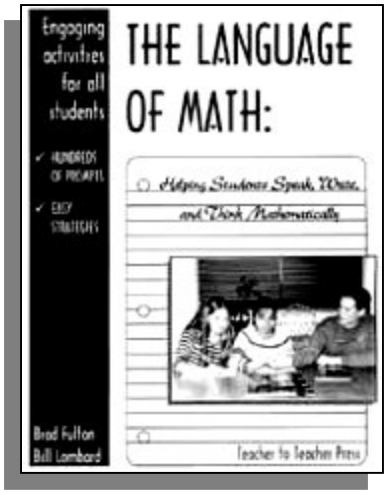
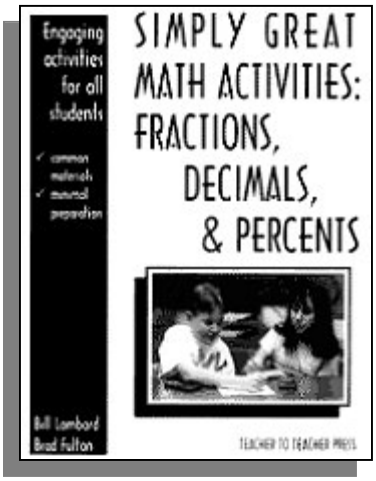
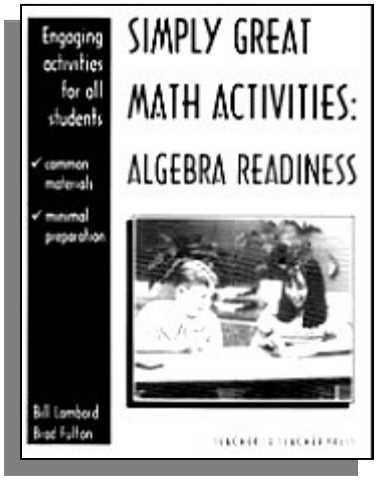


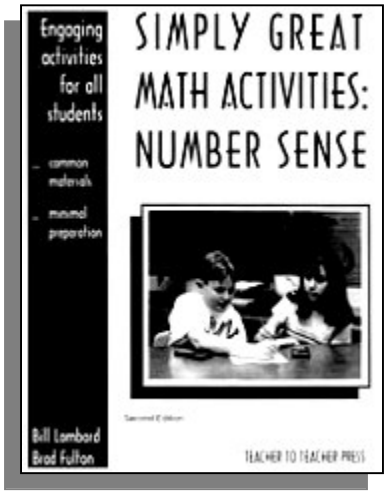
The Language of Math helps teachers create a classroom environment rich in mathematical thinking by showing them how to easily incorporate oral and written language into their math classes. Over 100 journal and discussion starters are included along with extensive instructions for making the most of your math time.



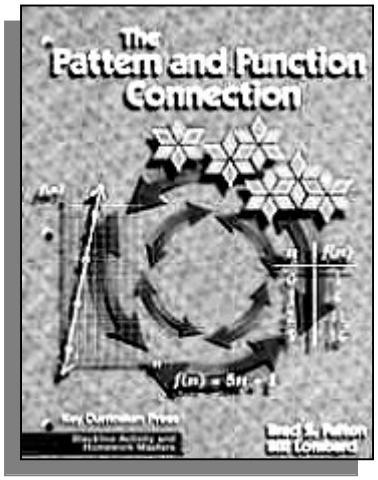
Here are a dozen unique and conceptual activities that will help your students add, subtract, multiply and divide fractions as well as connect them to decimal and percent representations. Both you and your students will love the novel and creative approach.



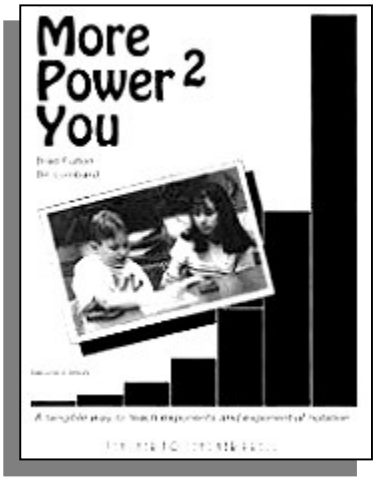
Teachers are raving about how effective these activities have been in their classrooms. Children as young as fourth grade and college students alike say that algebra is easy and makes sense because of this incredible approach.



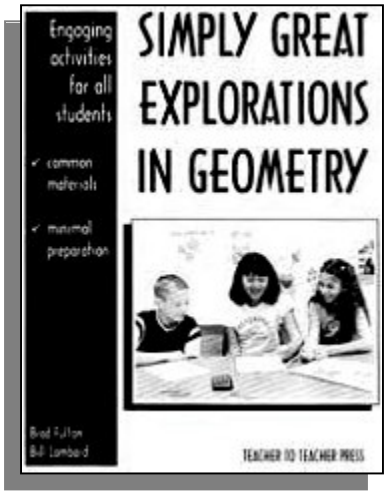
Students don't even think they are doing math sometimes because these activities are so fun and engaging, but they are developing rich and valuable number sense as they explore these eleven creative activities.



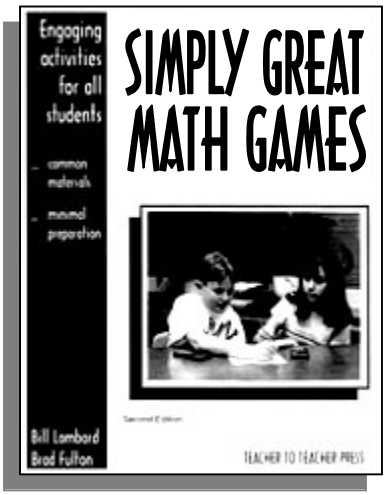
Our first book is still one of our most popular. Every teacher we talk to who has tried this approach to functions has been amazed at what their students have learned and accomplished. Over 150 pages of multiple representations of functions cover such concepts as slope, intercept, and function notation. Even elementary students have developed an understanding of functions with this book.



Exponents will finally make sense to your students after they participate in the unique activities found in this book. Both positive and negative exponents are demonstrated conceptually. Your students will even be able to explain *why* $n^0 = 1$.



Over one dozen geometry activities will excite your students as they discover the connections between geometry and fractions, decimals, percents, and even algebra. Area formulas, angle measurement, polygon attributes, vocabulary, and construction are covered.



A dozen engaging and educational games await you and your students in this creative and highly adaptable book. You'll find games that reinforce basic operations with whole numbers, fractions, decimals, and integers as well as algebraic skills. Game masters will serve a spectrum of grade levels and skill levels. Your students will beg for more!

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