

# Pick's Theorem, Pentominoes, and Dot Paper Bring the Standards Alive!

by  
Bill Lombard

Use dot paper to graph, find slope, area, symmetry, congruence, discover Pick's Theorem, solve pentomino problems and puzzles. All students can be successful!



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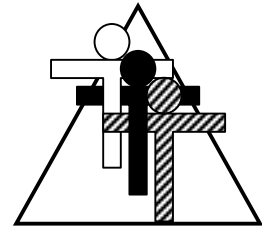
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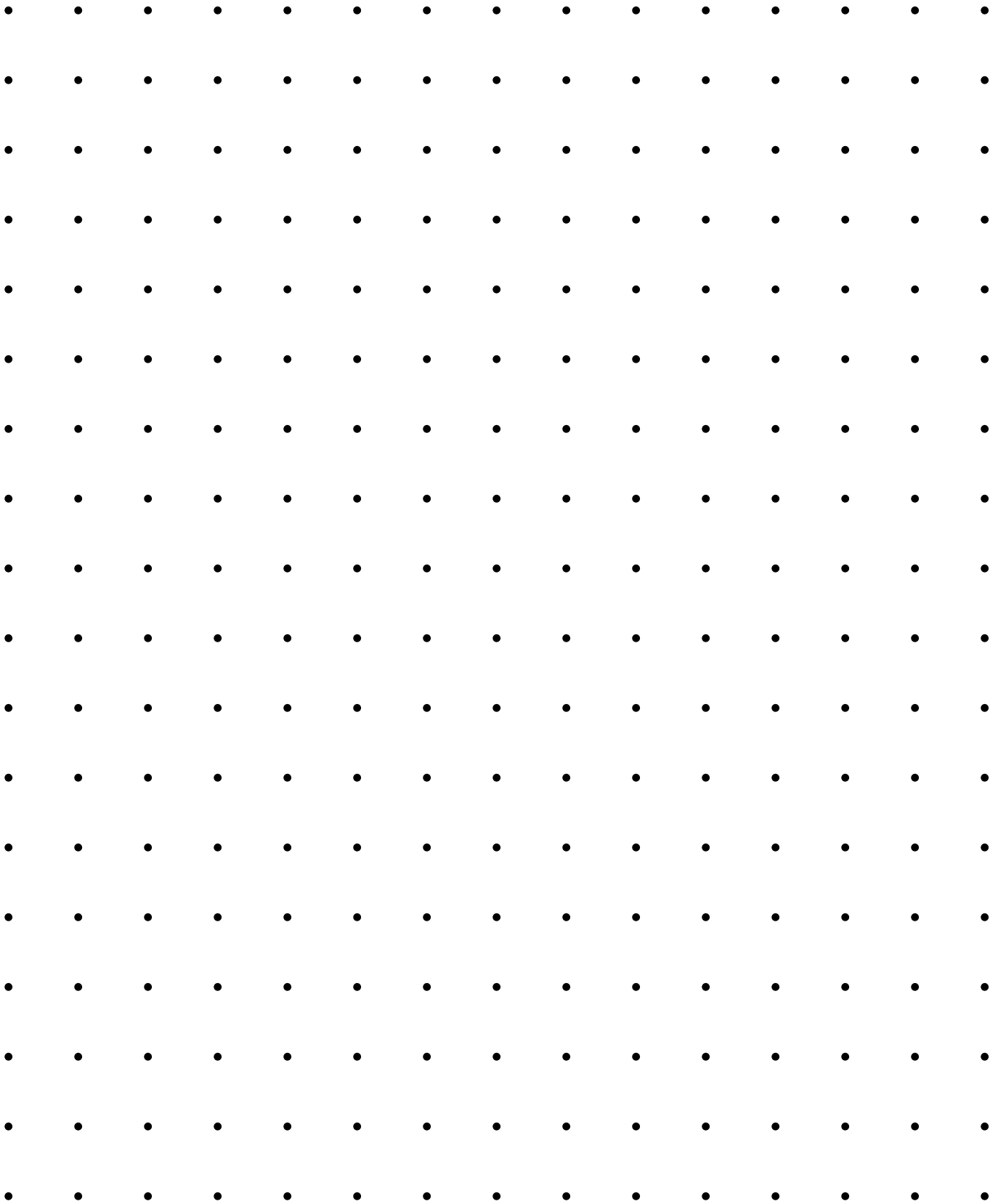
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“Building Mathematical Skill On a Foundation of Understanding”

$\frac{1}{2}$  inch dot paper



## ACTIVITY 7

# Pick's Pens

### Materials:

- ☒ Centimeter dot paper
- ☒ Pick's Theorem transparency

**Overview:** Students will study area and perimeter as they derive and use Pick's Theorem. This formula allows them to find the area of a polygon superimposed upon a square grid.

**Vocabulary:** theorem, area, perimeter

## PROCEDURE

### Skills:

- Finding area
- Deriving a formula
- Using a formula

posts	trees	area
6	1	3
6	2	4
6	3	5
6	4	6

- 1 Display pens one through four on the transparency master. Explain to the students that Farmer Pick wants to build some pens, but he doesn't want to dig postholes. He has decided to use the trees in his orchard as posts. He'll simply nail boards to them to build pens as shown on the transparency. He has hired the class to help him. He suspects that there is a way to predict the area of the pen based on the number of posts and interior trees. (Posts are trees that have boards nailed to them.) He would like them to use a t-table to analyze some pens he has already constructed to see if there is a pattern.

- 2 Pass out centimeter dot paper. Have the students sketch pens one through four and make a three-column t-table as shown in the margin. They should then record the posts, trees, and areas of the first four pens. Ask the class what pattern they see. (Adding a tree increases the area by one square unit.) They may also notice that the area is two more than the number of trees.

$$A = t + 2$$

posts	trees	area
4	0	1
5	0	1.5
6	0	2
7	0	2.5

- 3 Now reveal pens five through eight and ask them to add these to the t-table. What pattern do they notice? (adding a post adds only a half of a square unit.) Now there does not seem to be such an apparent relationship between the number of posts and the area. However if you cut the number of posts in half, the relationship is more easily seen. The area is half the number of posts minus one.

$$A = \frac{P}{2} - 1$$

NOTE: This activity is from *Simply Great Explorations in Geometry*, Teacher to Teacher Press

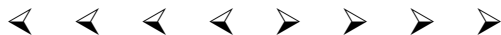
- 4 Now the first formula doesn't work; you can't add two to the number of trees (0) to get the area. Students may notice that they can add zero trees to the second formula and it still works:

$$A = \frac{P}{2} + t - 1$$

- 5 If a student does not notice this, you can point it out. This new formula also works for the first four pens.
- 6 Now reveal pens nine through twelve. Have the students verify that Pick's Theorem works by calculating the area of each pen and using the formula. Answers are shown here.
- 7 You may now wish to have the students design pens of their own and use Pick's Theorem to find the areas of them.
- 8 Your students may be interested to know that Pick's Theorem was named for Georg Alexander Pick. Born in Vienna, Austria in 1859, he published his theorem in 1899. Sadly, he died in 1943 in the Theresienstadt concentration camp.

$P/2$	area
2	1
2.5	1.5
3	2
3.5	2.5

posts	trees	area
12	4	9
10	2	6
12	1	6
13	3	8.5



### Journal Prompts:



- What effect does adding a post have upon the area of a pen?  
 What effect does adding a tree have upon the pen's area?  
 Explain how you know this is true.  
 How are area and perimeter related? Can you have two regions with the same perimeter and different areas or vice versa?

### Homework:

Students can use Pick's Theorem to check their work on other activities in this book. Examples include "Tangram Areas," and "Super Tangram," and the state areas in "Deriving Area Formulas." They will need to put a piece of the appropriate size dot grid underneath the diagram. Half-inch dot paper works well for the first two activities, and centimeter dot paper works for the states. Students must remember that when using the half-inch dot paper, the answer is expressed in square half-inches. There are four square half-inches in a square inch.

NOTE: This activity is from *Simply Great Explorations in Geometry*, Teacher to Teacher Press

### Taking a Closer Look:

Students may wish to design their own polygons and make up answer keys to them.

Ask students to find the largest area that can be included in pens of specific perimeters. What is the largest pen that has 16 fence posts? What if there are 22 fence posts? What if there are 17 posts? What is the fewest number of posts needed to encompass an area of 18 square units?

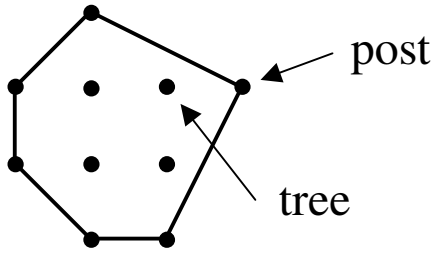
You may wish to have students write the areas of various polygons in an array as shown below. It is very easy to see then that increasing a post adds a half of a square unit of area to the pen, and increasing by a tree adds one square unit of area.

### Assessment:

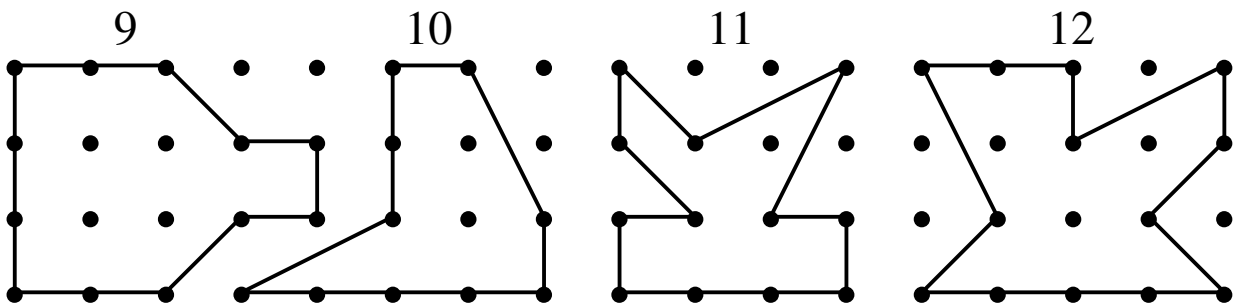
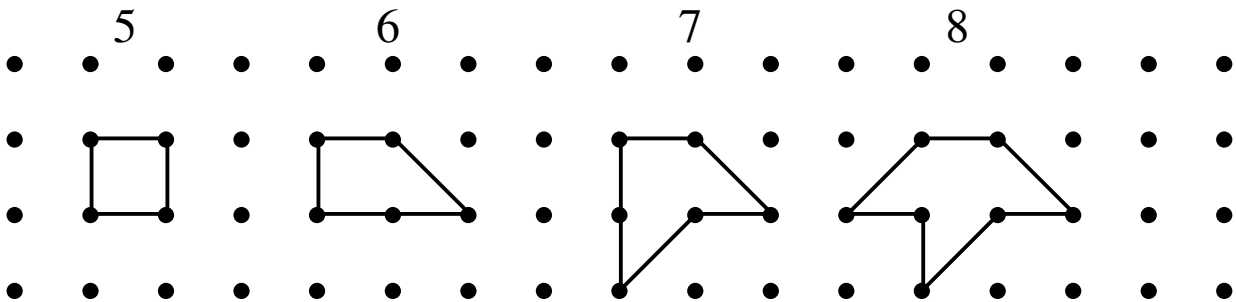
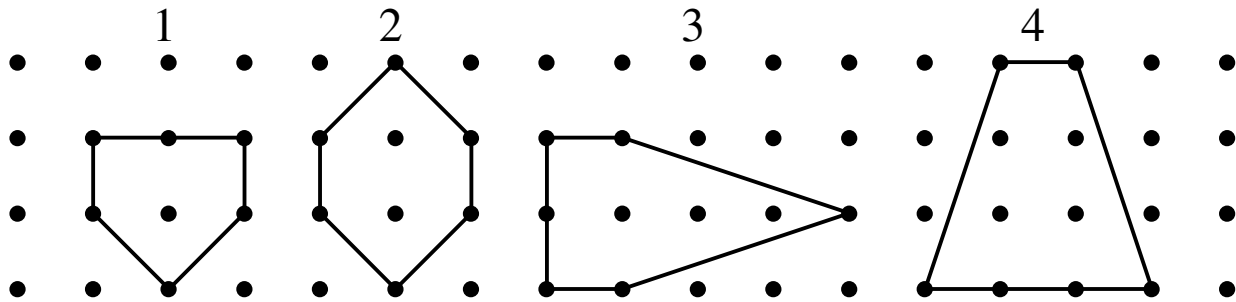
Use the t-tables in the margins on the preceding pages to check student answers. If you have students design their own polygons, they can check each other's answers. Alternatively, you can have them transfer their polygon to a transparency of dot paper for the class to solve.

		Posts				
		3	4	5	6	7
Trees	0	.5	1	1.5	2	2.5
	1	1.5	2	2.5	3	3.5
	2	2.5	3	3.5	4	4.5
	3	3.5	4	4.5	5	5.5

NOTE: This activity is from *Simply Great Explorations in Geometry*, Teacher to Teacher Press



# Pick's Pens



NOTE: This activity is from *Simply Great Explorations in Geometry*, Teacher to Teacher Press

## ACTIVITY 7

# Squares on a Geoboard

### Materials:

- geoboard paper
- transparent geoboard or the transparency provided on page 37
- geoboards

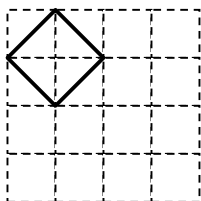
**Overview:** Students will explore the different ways to build squares on a geoboard. They can use the Pythagorean Theorem to prove which squares can be built on the grid and which squares cannot.

**Vocabulary:** square, square root, Pythagorean Theorem

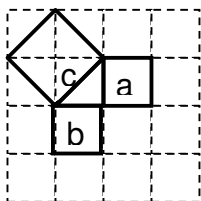
## PROCEDURE

### Skills:

- Using the Pythagorean Theorem
- Finding areas
- Using square numbers



A square with an area of two square units.



- 4 Pass out geoboard paper from page 36 and geoboards if you are using them. Ask the students what size squares can be built on a geoboard. Have them build and sketch each solution. Then let students show their solutions on the overhead projector using a transparent geoboard or the transparency from page 37.
- 5 Most students will easily see four solutions, the 1x1 square, the 2x2, the 3x3, and the 4x4. These have areas of 1, 4, 9, and 16 square units respectively. However, some student may notice that you can also build a square with an area of two square units as shown in the margin. If no one shows this solution, offer it. This should open the door to other possible solutions to this problem. Let the students try to find the others. All eight solutions are shown below under "Assessment."
- 6 You can use the Pythagorean Theorem to verify the areas of the solutions that do not have a horizontal base. In the example below, look at the triangle between the three squares. Sides a and b are both one unit in length. Using the Pythagorean Theorem gives us:

$$a^2 + b^2 = c^2$$

$$1^2 + 1^2 = c^2$$

$$1 + 1 = c^2$$

$$2 = c^2$$

$$\sqrt{2} = c$$

Therefore, the rotated square has an area of  $\sqrt{2}^2$  or 2.

This activity is from *Simply Great Explorations in Geometry*, Teacher to Teacher Press

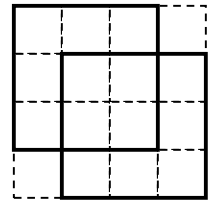


**Journal Prompts:**



Explain why a square with an area of twelve square units cannot be made on a geoboard. (Twelve is not the sum of two squares.)

Explain why a square with an area of thirteen square units cannot be made on a geoboard. (Thirteen is the sum of  $2^2 + 3^2$ . However, it won't fit on a 4x4 geoboard since  $2 + 3 > 4$ .)



Two 3x3 squares can overlap.

**Homework:**



Give the students 1/4-inch grid paper and so they can outline some 10x10 geoboards. Have them show all squares that can be constructed on the 10x10 board.

**Taking a Closer Look:**



You may wish to have students explore a different problem on the geoboard. Ask them how many total squares can be made on the geoboard. Sliding a shape and overlapping are allowed as shown in the margin. An interesting pattern emerges as shown in the t-table in the margin.

This problem can also be applied to an 8x8 or a 10x10 geoboard.

size	quantity
1x1	16
2x2	9
3x3	4
4x4	1
Total	30

**Assessment:**



Here are the solutions for a 4x4 geoboard:

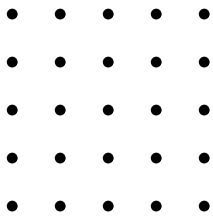
Area = $1\text{un}^2$ 	Area = $2\text{un}^2$ 	Area = $4\text{un}^2$ 	Area = $5\text{un}^2$ 
Area = $8\text{un}^2$ 	Area = $9\text{un}^2$ 	Area = $10\text{un}^2$ 	Area = $16\text{un}^2$ 

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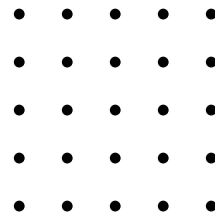
# Squares on a Geoboard

## 4x4 Dot Grids

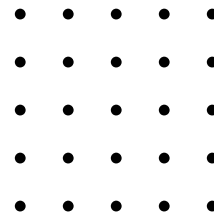
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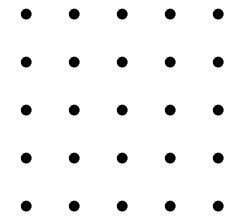
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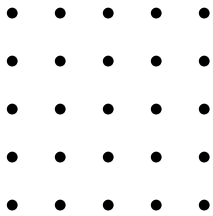
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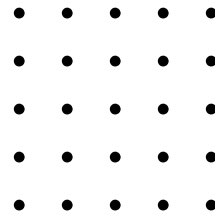
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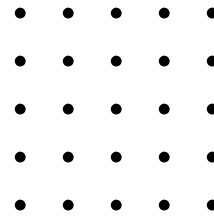
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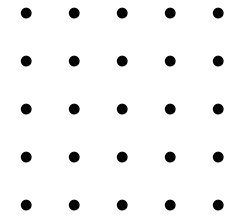
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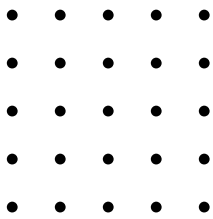
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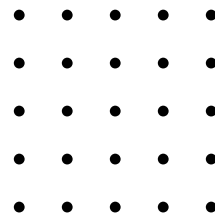
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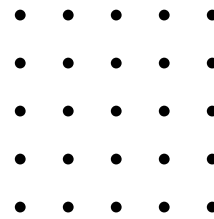
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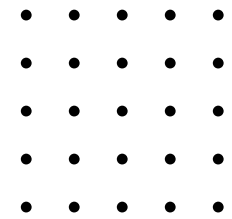
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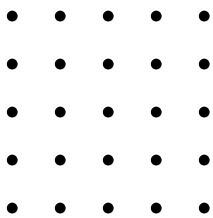
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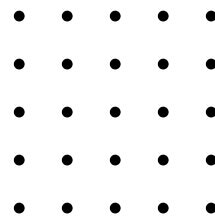
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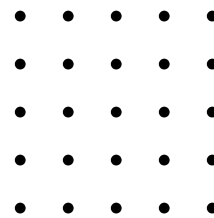
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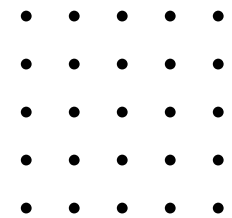
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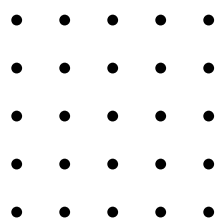


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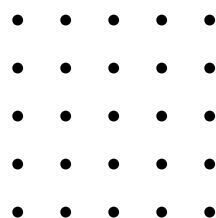
# Slope on a Geoboard

## 4x4 Dot Grids

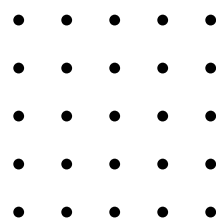
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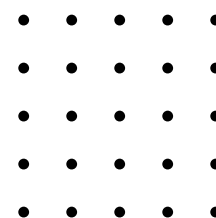
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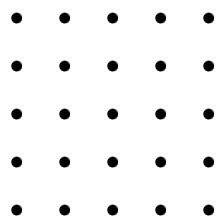
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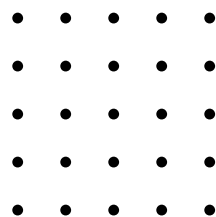
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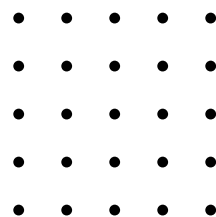
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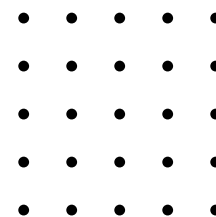
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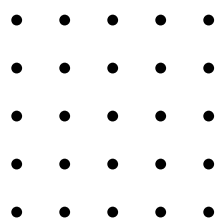
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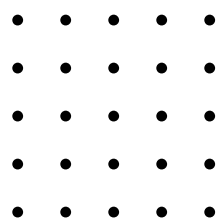
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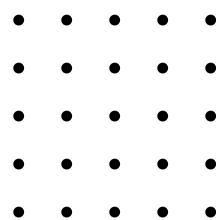
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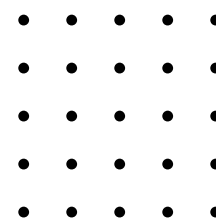
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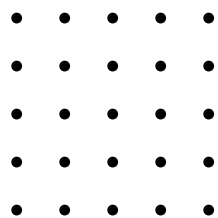
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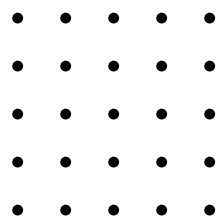
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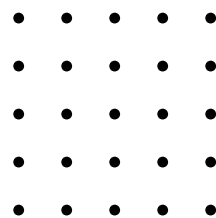
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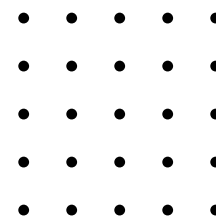
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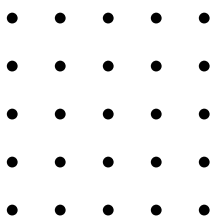


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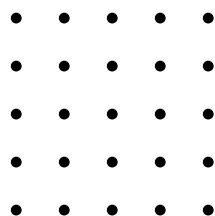
# Triangles on a Geoboard

## 4x4 Dot Grids

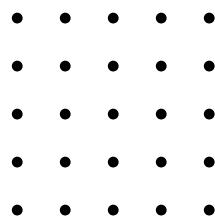
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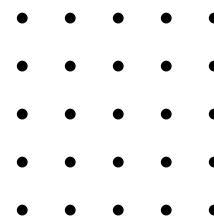
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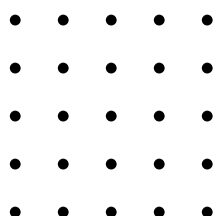
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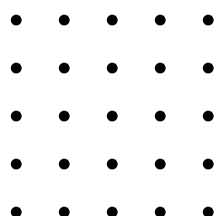
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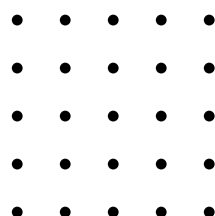
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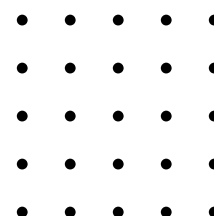
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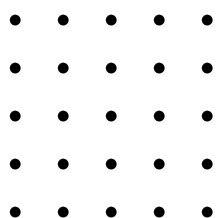
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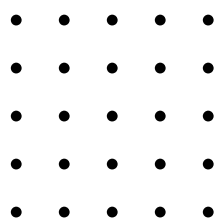
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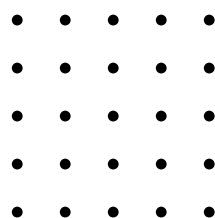
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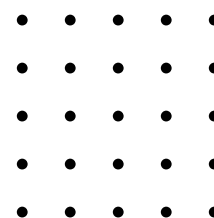
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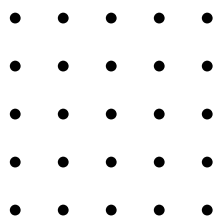
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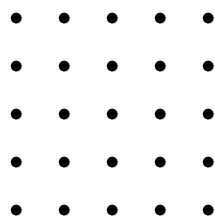
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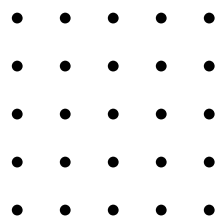
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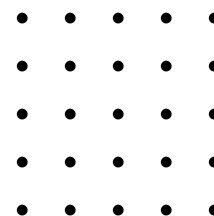
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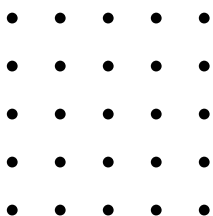


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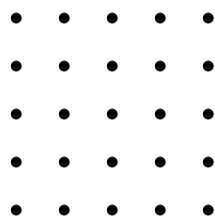
# Quadrilaterals on a Geoboard

## 4x4 Dot Grids

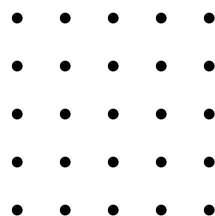
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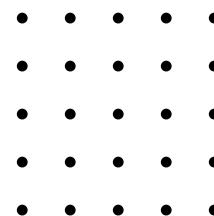
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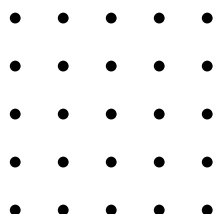
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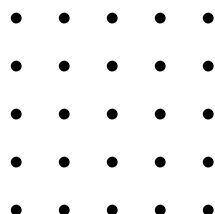
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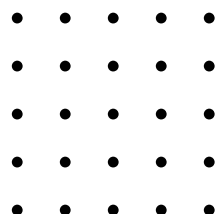
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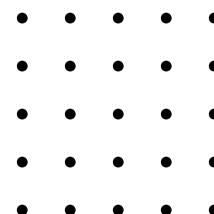
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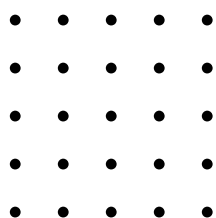
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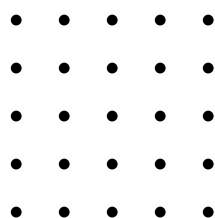
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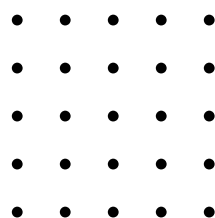
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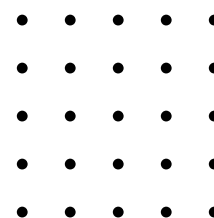
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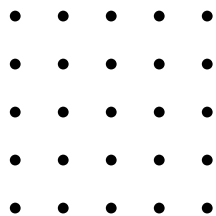
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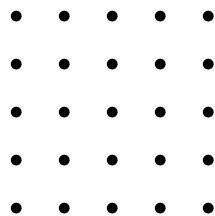
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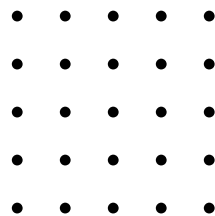
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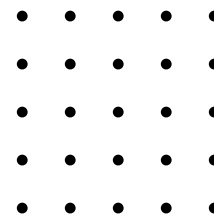
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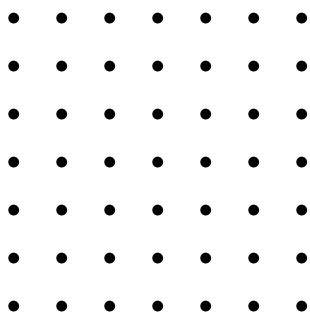


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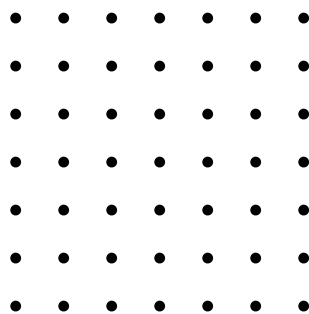
# Squares on a Geoboard

## 6x6 Dot Grids

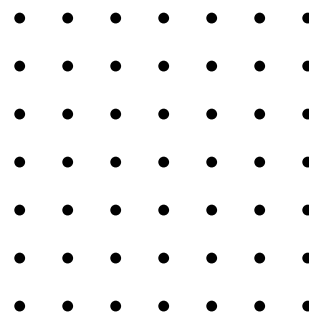
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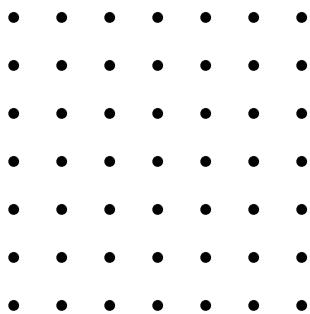
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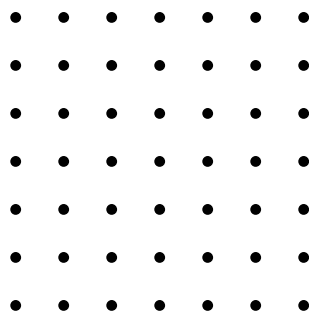
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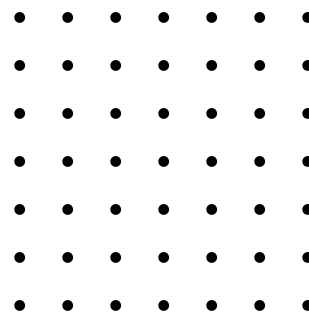
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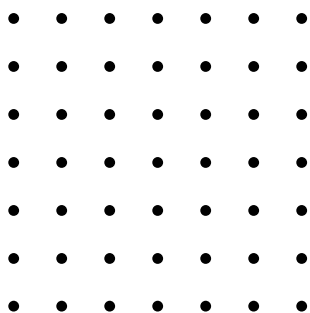
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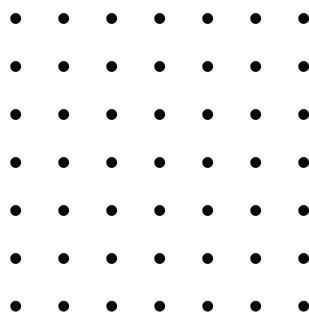
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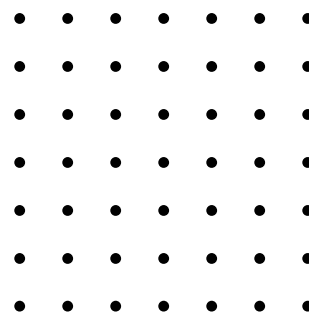
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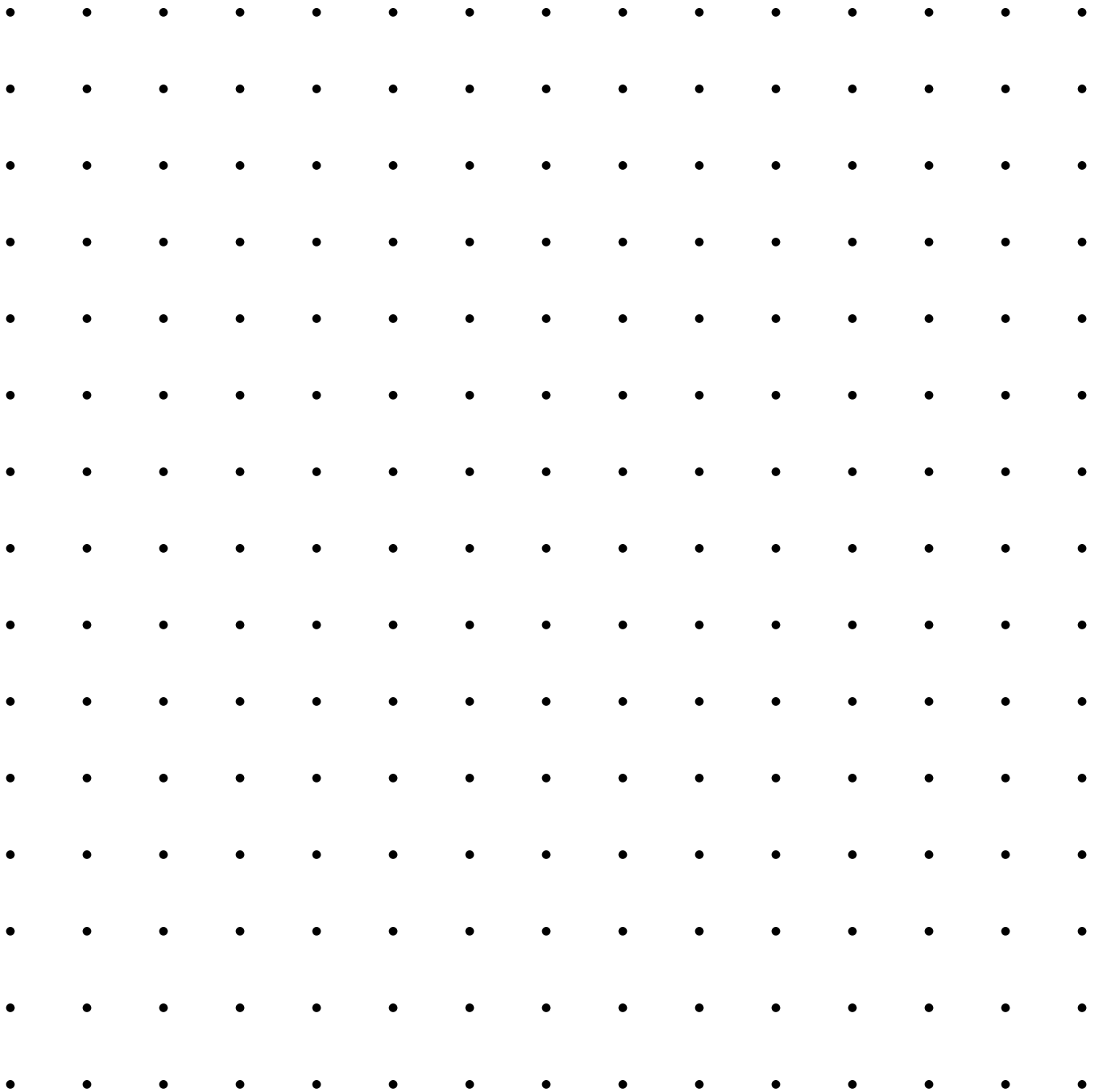
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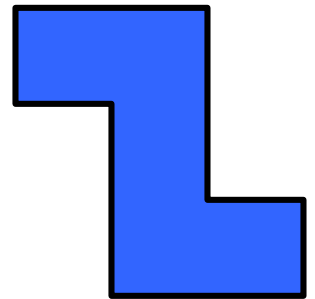
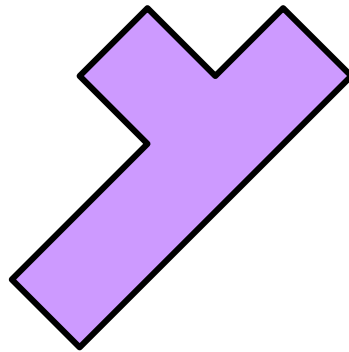
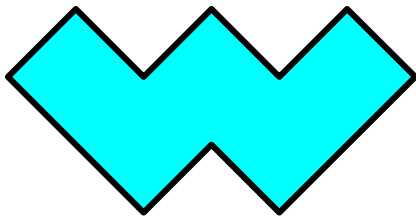
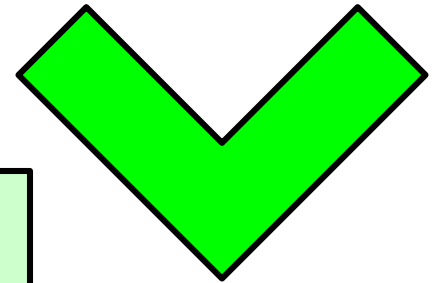
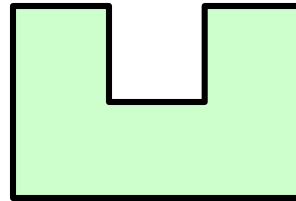
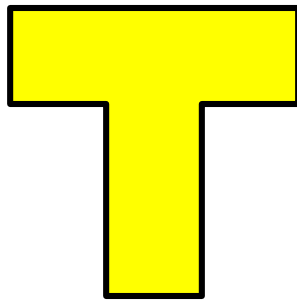
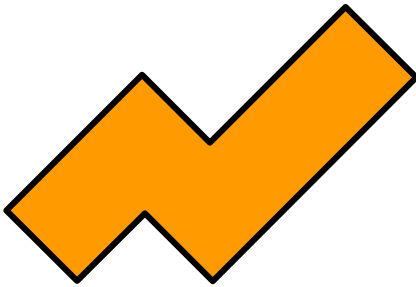
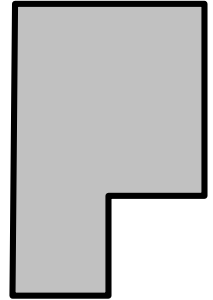
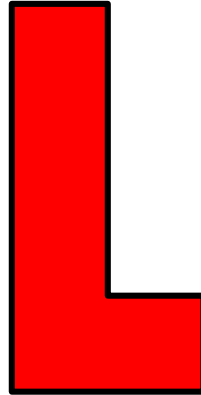
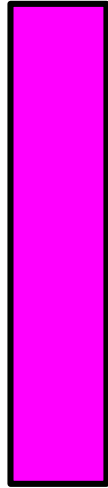
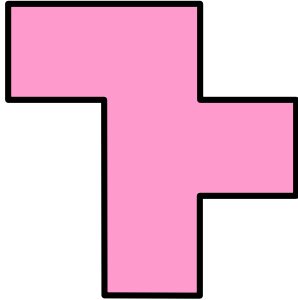
# Pentomino Discovery

Pentominoes are five squares joined on their edges.  
How many pentominoes can you find?



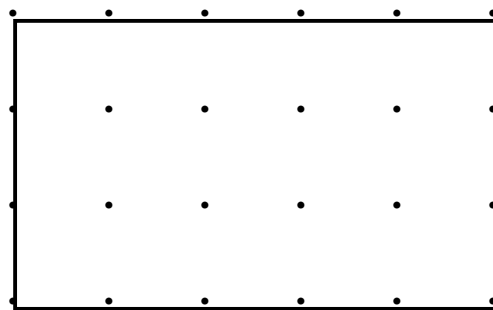
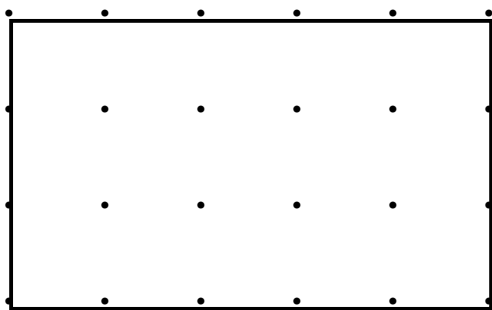
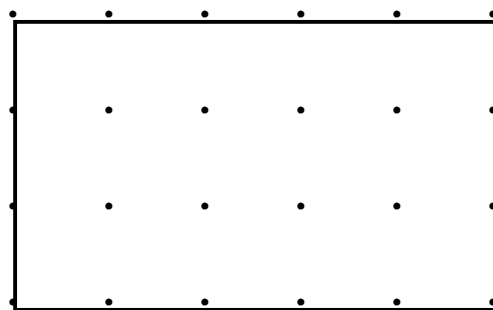
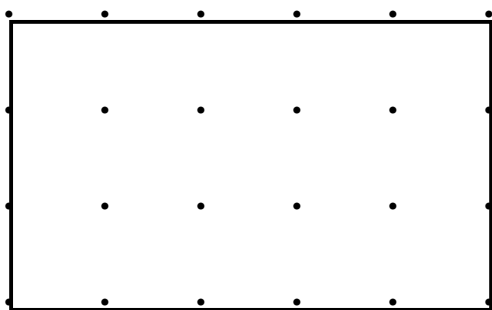
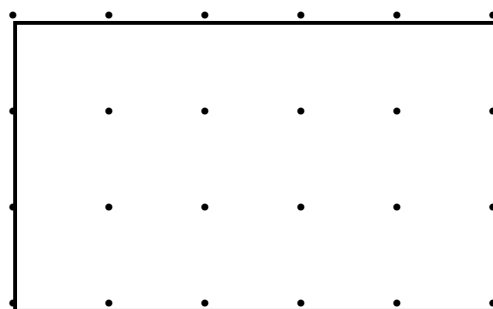
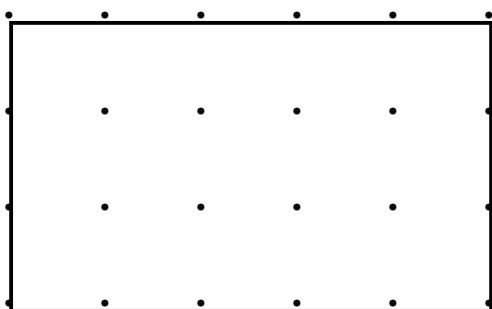
# Pentominoes

F, I, L, P, N, T, U, V, W, X, Y, Z



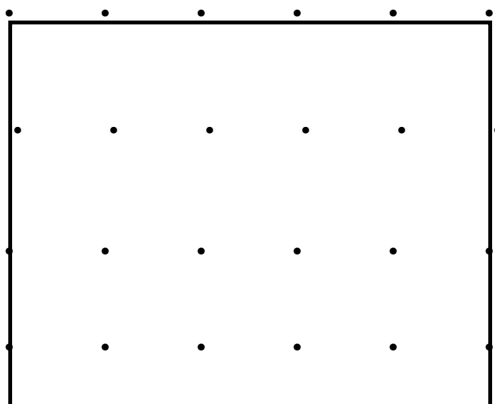
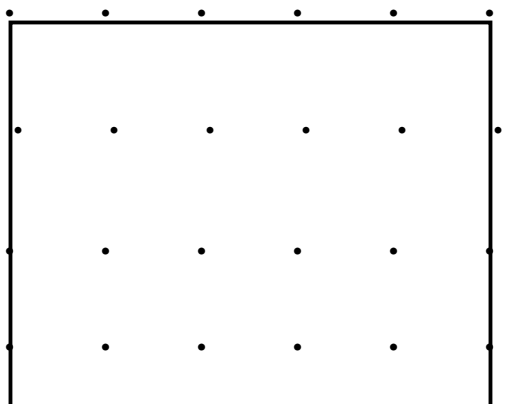
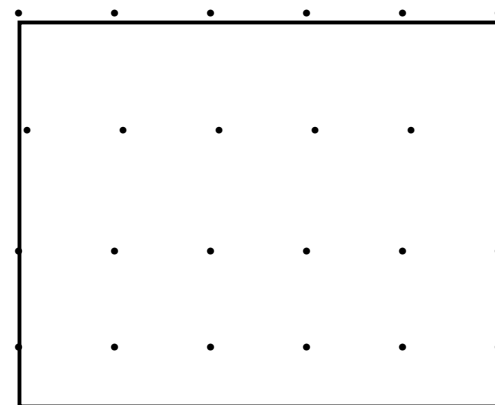
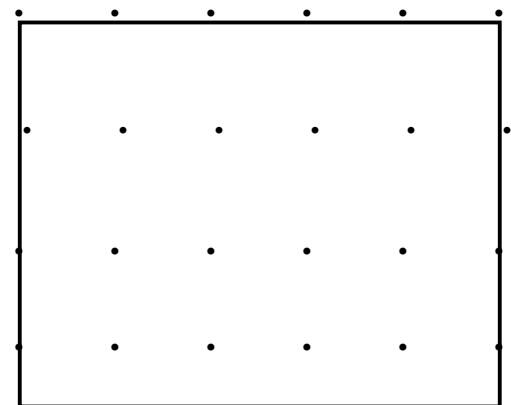
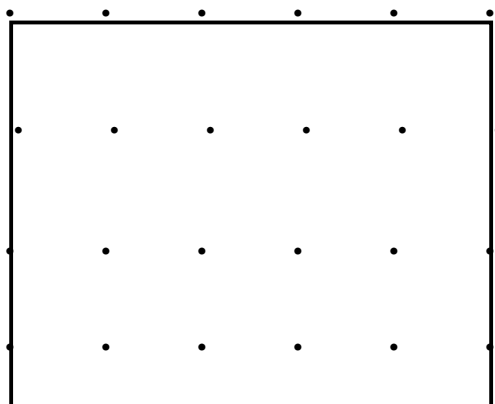
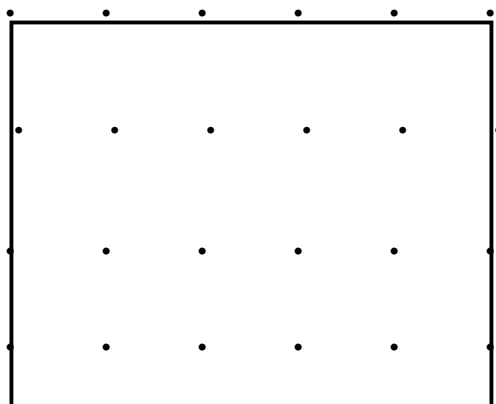
# Pentomino Puzzles

Use three pentominoes to make a rectangle. Use the grids below to sketch your work. How many ways can you do this? Which pentominoes cannot be used?



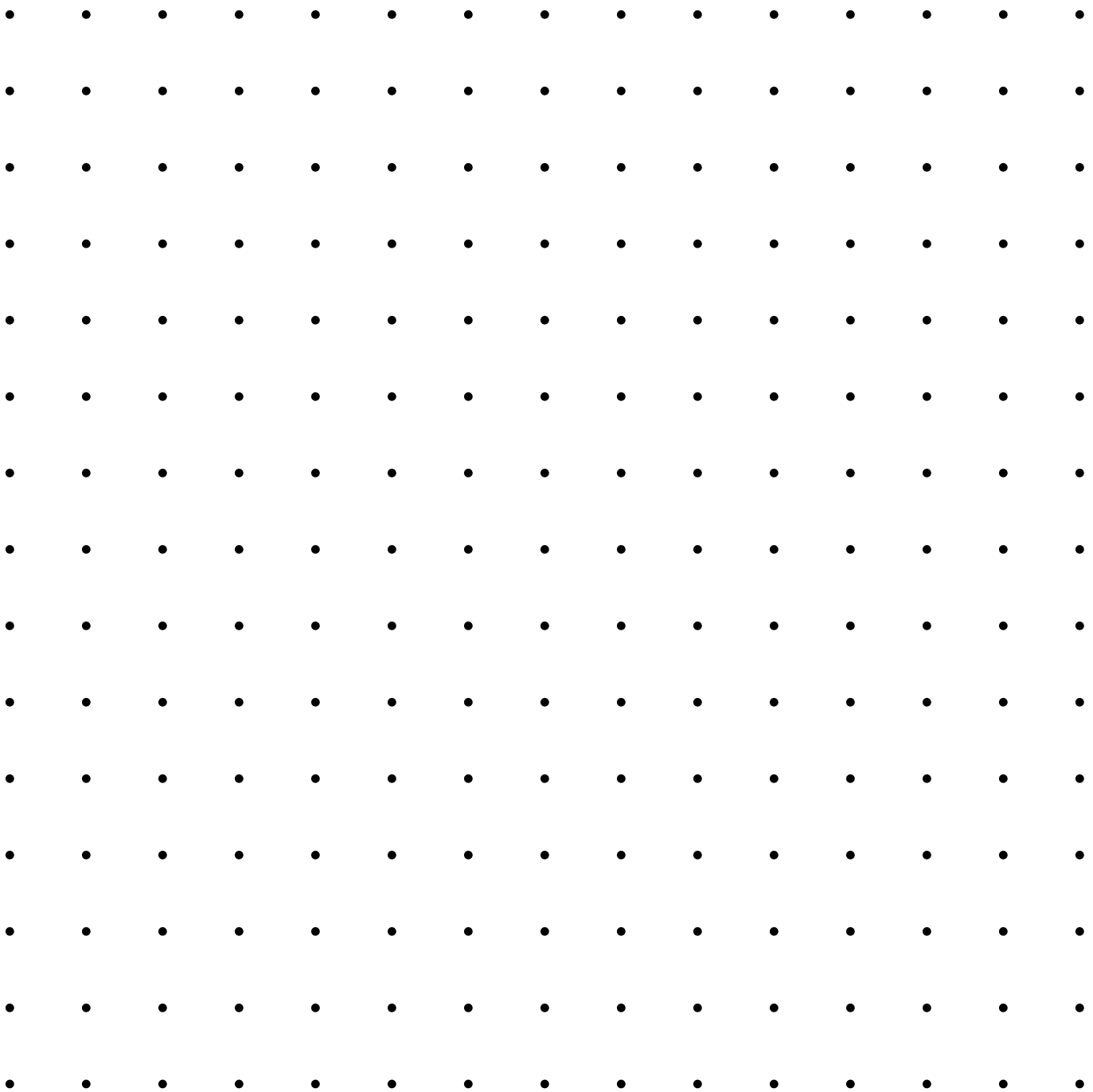
# Pentomino Puzzles

Use four pentominoes to make a rectangle. Use the grids below to sketch your work. How many ways can you do this? Which pentominoes cannot be used?



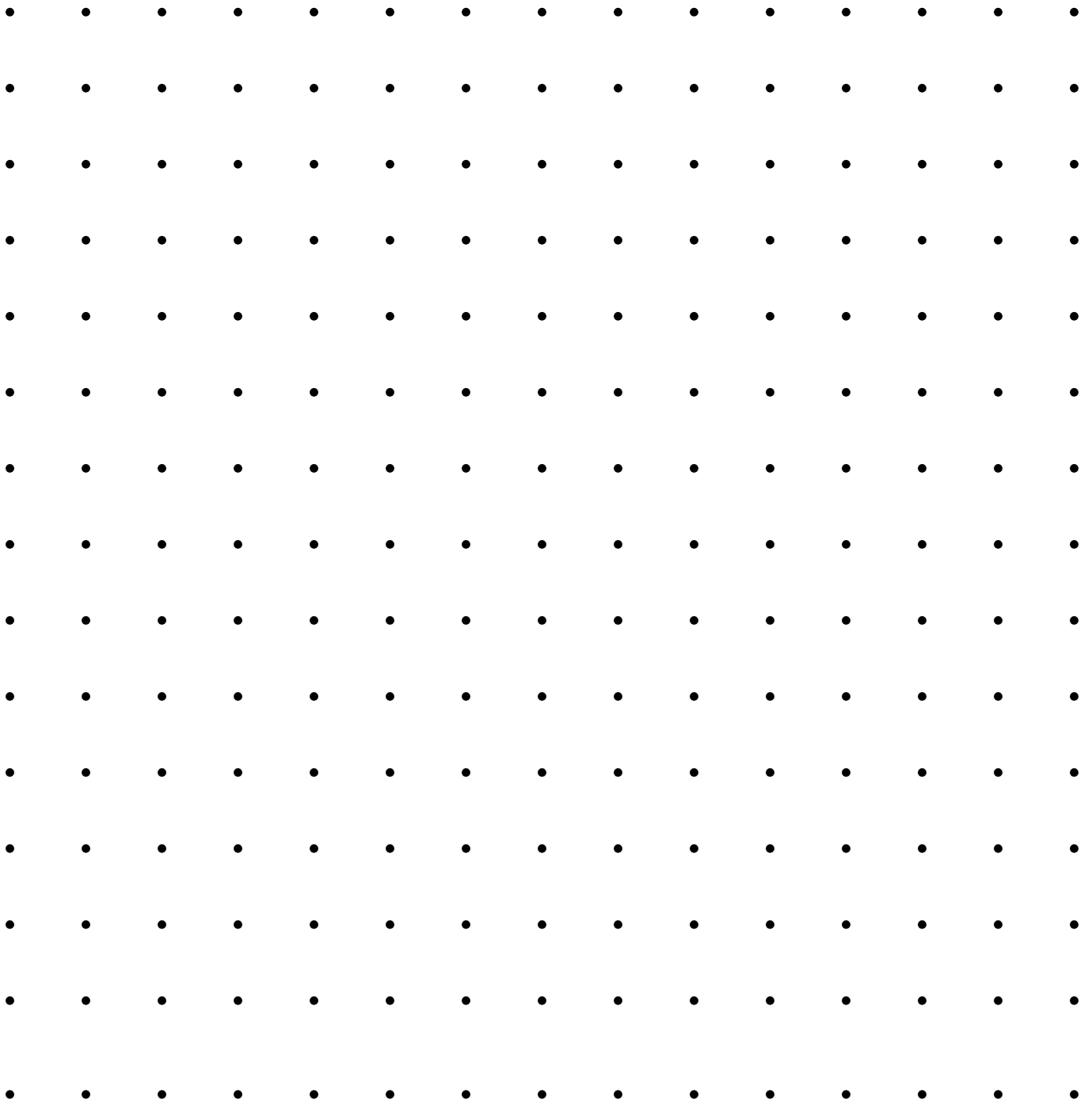
# Pentomino Boxes

Sketch below the pentominoes that will form an open-topped box when folded. You will be making five sides of a cube.



# Pentomino Tessellations

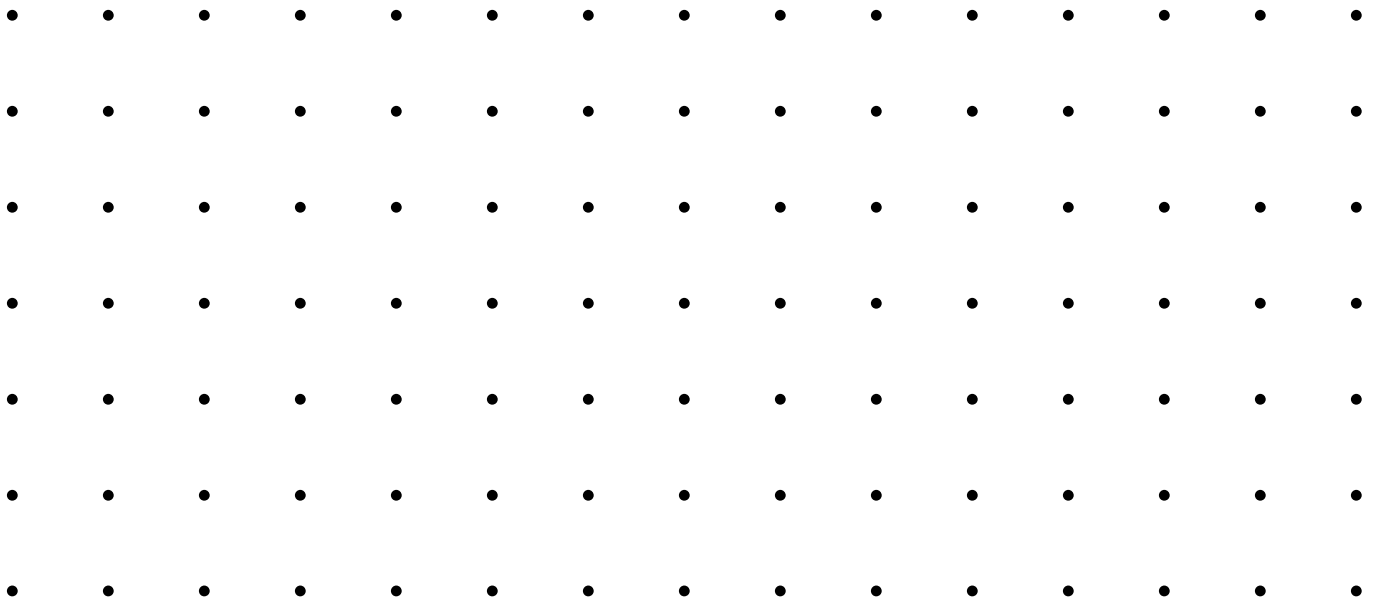
Tile the grid below using the same pentomino.  
You may need to rotate or reflect the pentominoes to  
make them tessellate.



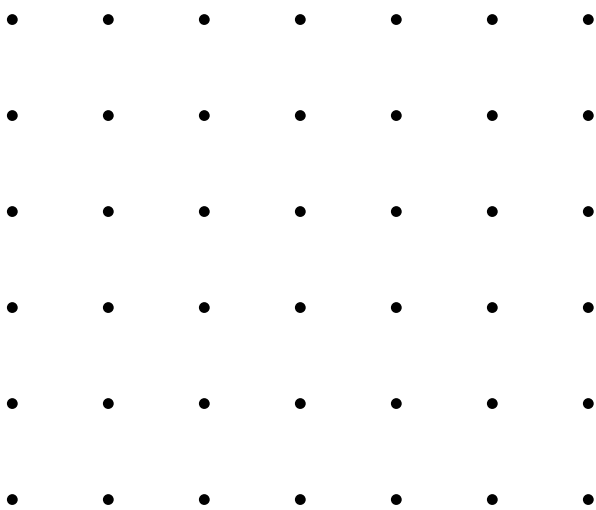
# Pentomino Symmetry

List the pentominoes below that have symmetry.

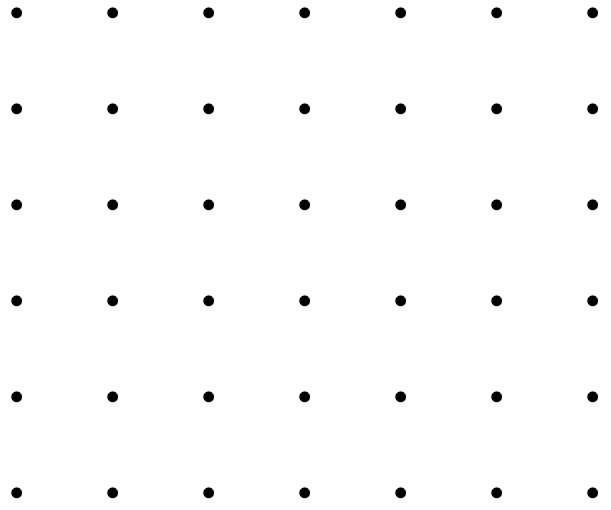
Line symmetry only:



Point symmetry only:

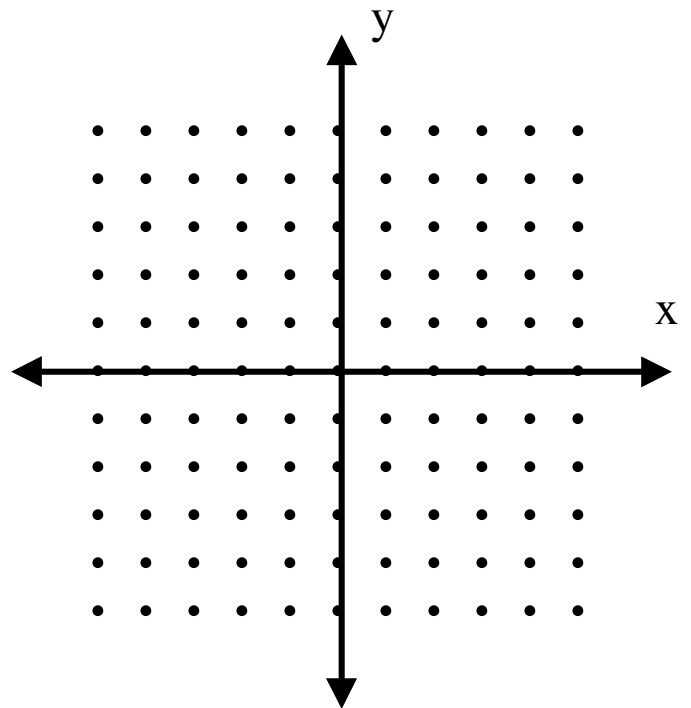
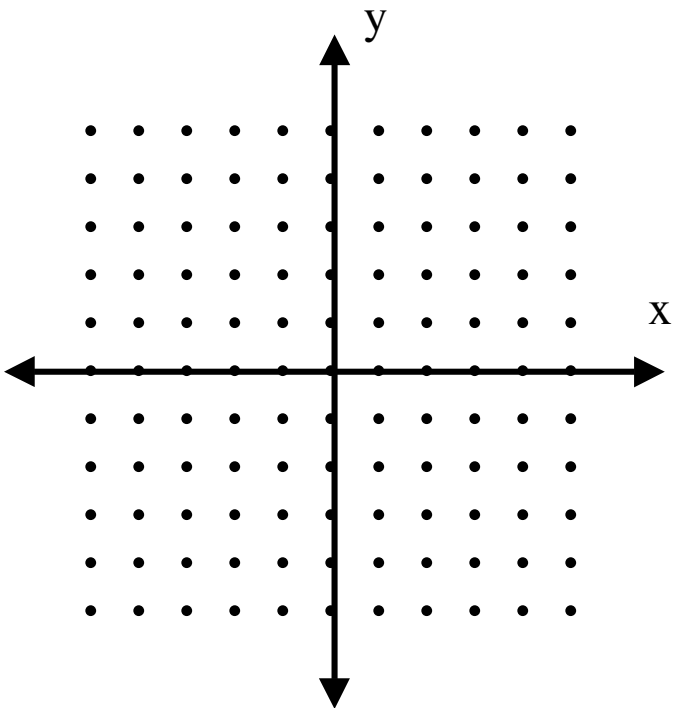
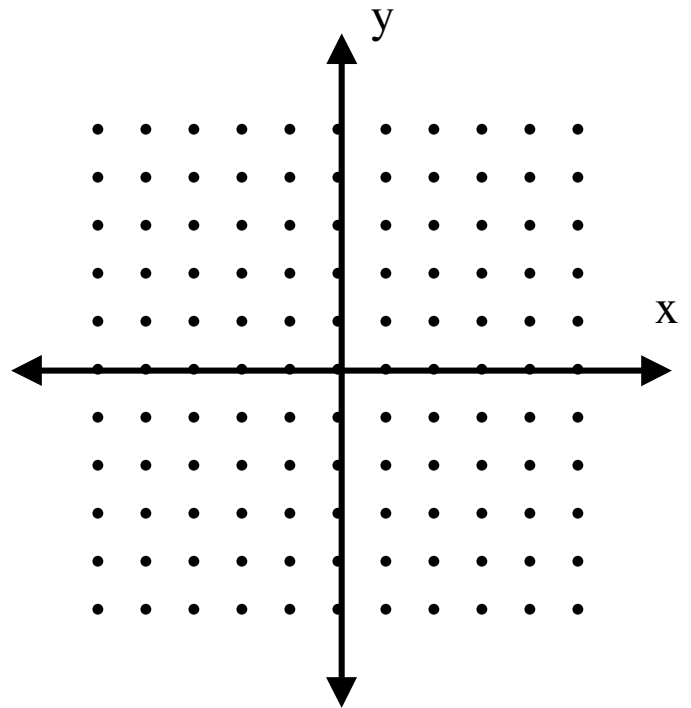
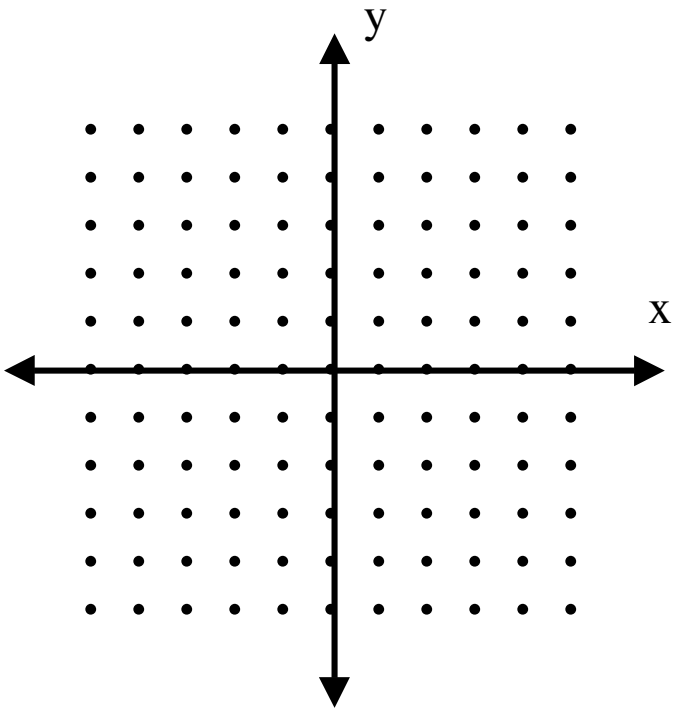


Point and line symmetry:



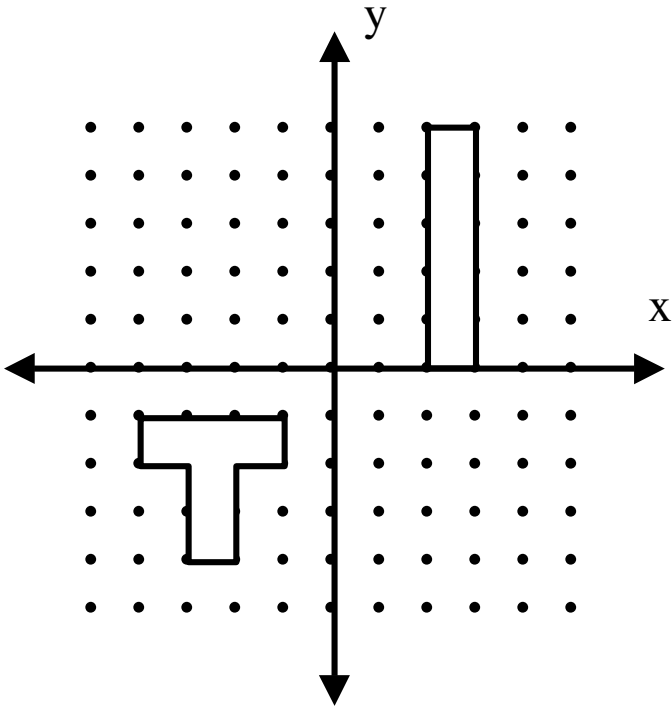
# Pentomino Plotting

Place all twelve pentominoes on the grids below, and label the coordinates of the corners (vertices).



# Pentomino Equations

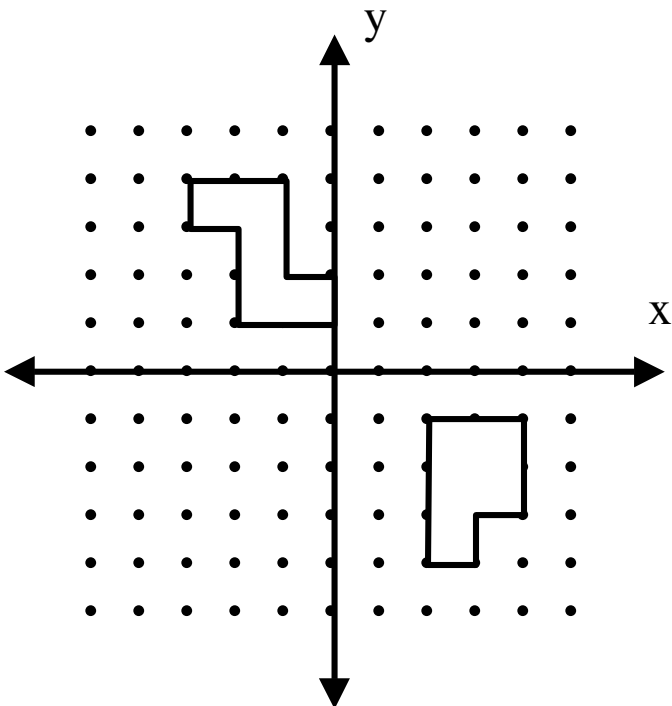
Give the equations for the line segments forming the pentominoes on the grids below.



I. \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

T. \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

Z. \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_



P. \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_  
 \_\_\_\_\_

# Twelve Dots and Five Lines

Draw five connected line segments through all twelve dots; once you start drawing, you can't lift your pencil from the paper until you're done. It's okay to visit a dot more than once and okay to have lines that cross. No line segments can be outside the rectangular region made by the dots.

1

o o o o  
o o o o  
o o o o

2

o o o o  
o o o o  
o o o o

3

o o o o  
o o o o  
o o o o

4

o o o o  
o o o o  
o o o o

5

o o o o  
o o o o  
o o o o

6

o o o o  
o o o o  
o o o o

7

o o o o  
o o o o  
o o o o

8

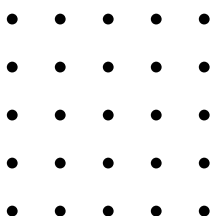
o o o o  
o o o o  
o o o o

# Dotty

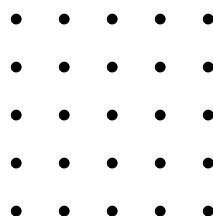
## A Two Player Game

Players take turns drawing line segments connecting one dot to the next in any direction except diagonally. The player drawing the segment that completes a square puts his/her initial in the square and draws another segment. The winner is the one with the most squares.

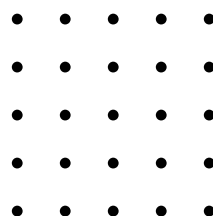
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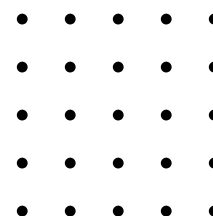
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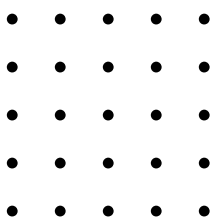
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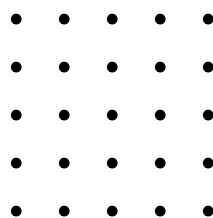
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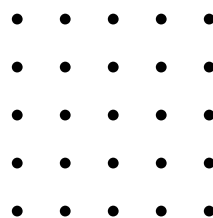
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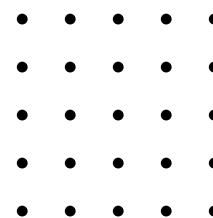
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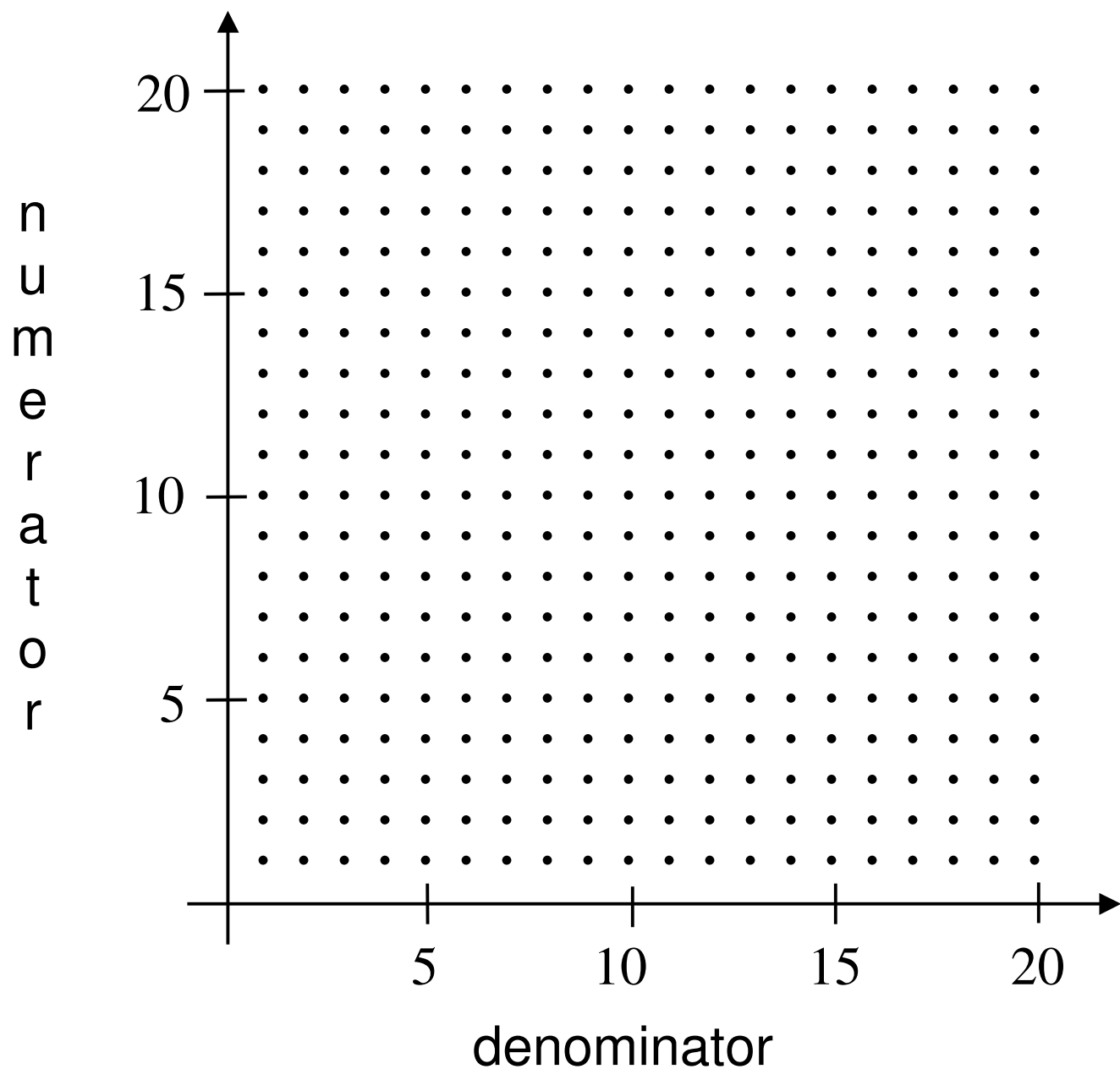
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8



# Fraction Finder



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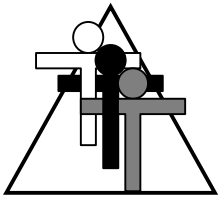
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Happy surfing!



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*Teacher to Teacher Press*

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- ◆ Educators
- ◆ Authors
- ◆ Seminar leaders
- ◆ Teacher trainers
- ◆ Conference speakers



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Known throughout the country for motivating and engaging teachers and students, Brad and Bill have authored over ten books that provide easy-to-teach yet mathematically-rich activities for busy teachers. In addition, they have co-authored six teacher training manuals full of activities and ideas that help teachers who believe mathematics must be both meaningful and powerful.

### **Seminar leaders and trainers of mathematics teachers**

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- ◆ Lead trainers for summer teacher training institutes
- ◆ Trainers/consultants for district, county, regional, and national workshops

### **Authors and co-authors of mathematics curriculum**

- ◆ *Simply Great Math Activities* series: five books covering all major strands
- ◆ *Math Discoveries* series: bringing math alive for students in middle schools
- ◆ Teacher training seminar materials handbooks for elementary, middle, and secondary school

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Wayne Dequer, 7th grade math teacher

*"The high-energy, easy-to-follow handouts were clear. The instructors were great!"*

DeLinda Van Dyke, middle school teacher

## References and sources for Pick's Theorem, Pentominoes, and Dot Paper

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